Quantum Black Holes in de Sitter Space-time

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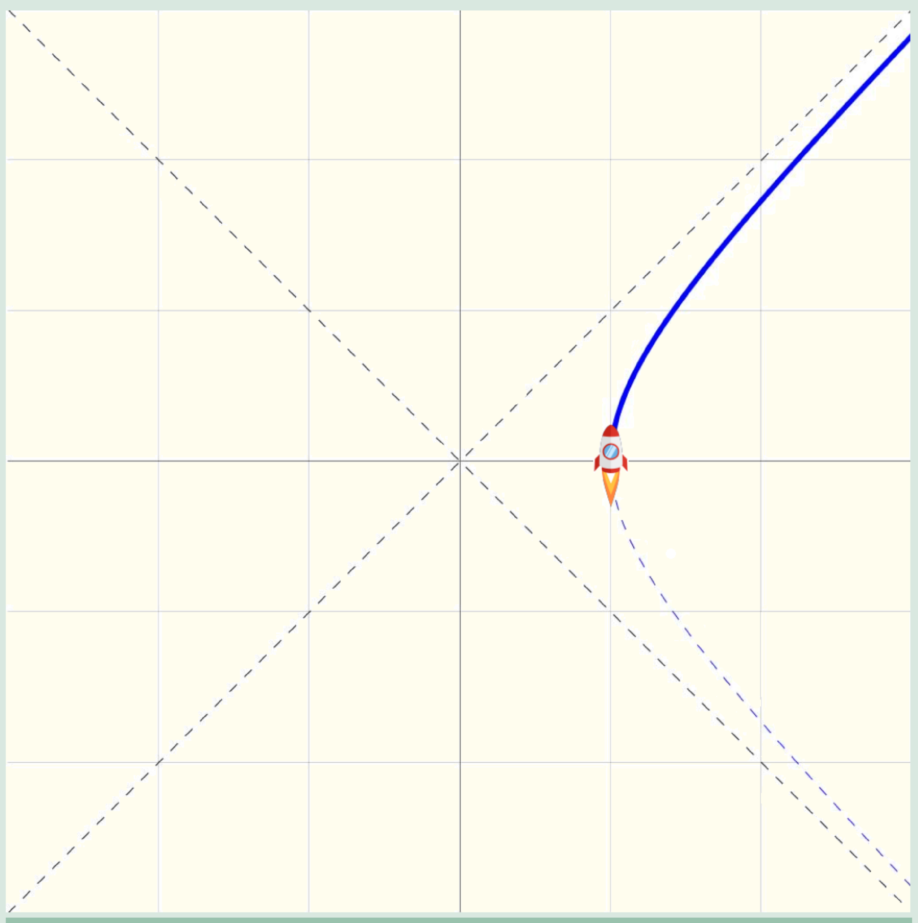
Introduction

The two pillars on which modern theoretical physics stands on, are General Relativity (GR) and Quantum Field Theory (QFT). On one hand these theories are so fundamentally different, that we have been unsuccessful at creating a theory of quantum gravity. On the other hand, these two theories, both being field theories, are compatible at a semi-classical level. This project studied how quantum fields propagating in different space-times can lead to many interesting results, particularly to do with black holes. First the Unruh effect [3] was studied to introduce how QFT can lead to particle creation. Next building on this, Hawking's original paper [2] on Hawking radiation was studied to see how this could be extended to black holes. Finally Raphael Bousso and Stephen Hawking's 1997 paper "(Anti-)Evaporation of Schwartzchild-de Sitter Black Holes" [1] was studied to see how black holes could evolve in the presence of quantum fields.

The Unruh Effect



Accelerating observer in a Minkowski diagram approaching $v = c$.

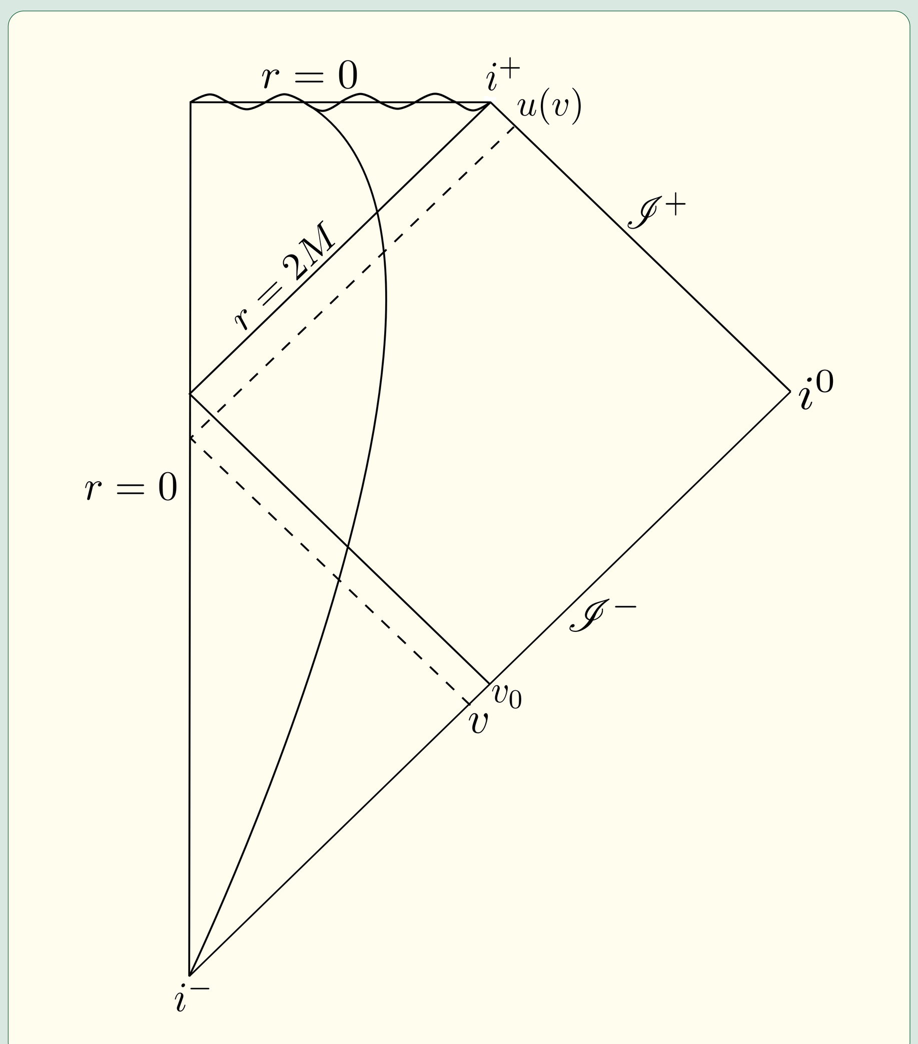
From Einstein's theory of Special Relativity we are familiar with the laws of physics being the same in every inertial frame. What happens if we betray Einstein and compare an inertial observer with a non-inertial (accelerating) frame; what affects will we see? It can be shown with some relatively simple QFT that a constant accelerating observer with acceleration a, will see the vacuum of a stationary observer to be full of particles.

This comes from the fact that if you examine a scalar Klein Gordon field in the co-ordinates of the stationary (labeled with c) and accelerating (labeled with b) observers you find that the creation and annihilation operators obey the following relation:

$$b_k^{(1)} \propto \left(c_k^{(1)} + e^{-\frac{\pi\omega}{a}} c_{-k}^{(2)\dagger}\right)$$

This means that when the accelerating observer acts on the vacuum in the frame of the stationary observer, the result is the creation of a particle due to the presence of $c^\dagger$. In fact if one carries out a formal calculation of the number of particles seen by the accelerating observer, one finds a Planck black body spectrum with a temperature $T = \frac{a}{2\pi}$:

Hawking Radiation



Penrose diagram of collapsing matter forming a Schwartzchild black hole

The Unruh affect, while in flat spacetime teaches us the most important lesson of QFT in curved spacetime, that being that the idea of the "vacuum" and "particles" are observer dependent quantities. Hawking Radiation comes from similar considerations; we have two different notions of a vacuum that arise, not just because one observer is accelerating, but due to the *geometry of spacetime.* When we relate the co-ordinates near the black hole to those in far away in the past, we find again a Black body spectrum, this time with temperature $T = 1/8\pi M = \frac{\kappa}{2\pi}$. Where κ is the *surface gravity.*

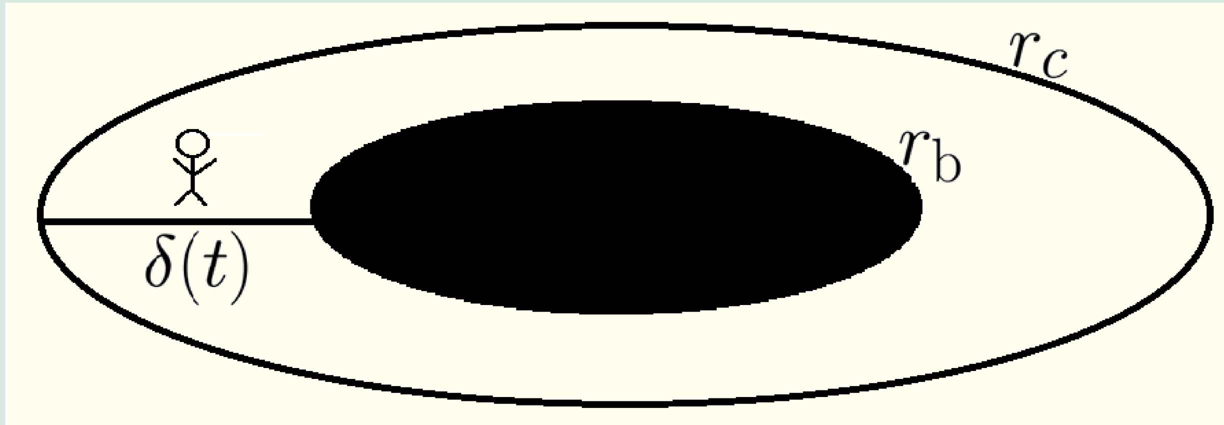
The idea of pair production at the horizon comes from the correlation of outgoing and ingoing spectra, $\implies$ occupation numbers are *entangled.*

Anti evaporation of certain black holes

If we extremise the Einstein Hilbert action with a cosmological constant we find the following Schwarzschild-de Sitter (SdS) metric:

$$ds^2 = -\left(1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2}dr^2 + r^2 d\Omega_2$$

In their paper Bousso and Hawking analyze the stability of this black hole.



Black hole near maximal size of cosmological horizon

Close to this limit the above SdS metric can be written in the form:

$$ds^2 = e^{2\rho}\left[-dt^2 + dx^2\right] + e^{-2\phi} d\Omega_2$$

With specific ρ and ϕ. Using this metric we can integrate out the an angular co-ordinates to obtain a two dimensional model:

$$S = \frac{1}{16\pi}\int d^2x\sqrt{-\tilde{g}}e^{-2\phi}\left[\tilde{R} + 2e^{2\phi} + 2(\tilde{\nabla}\phi)^2 - 2\Lambda - \frac{1}{2}\sum_{i=1}^{N}(\tilde{\nabla}f_i)^2\right]$$

Where we have introduced N scalar fields f as the Large N limit is needed to have evaporation contribute. To take quantum effects into account the scale-dependent part of the one-loop effective action for the N dilaton coupled scalars are added to this action. This introduces non-local terms which can be rendered local by the addition of an additional field Z:

$$S_{\text{tot}} = \frac{1}{16\pi}\int d^2x\sqrt{-\tilde{g}}\left[\left(e^{-2\phi} - \frac{N}{3}(Z + w\phi)\right)\tilde{R} - \frac{N}{6}(\tilde{\nabla}Z)^2 + 2 \right.$$
$$\left. +2e^{-2\phi}(\tilde{\nabla}\phi)^2 - 2\Lambda\, e^{-2\phi}\right]$$

The equations of motions of the fields can then be solved for and the dynamics of the black hole horizons determined from them. The classical case $N = 0$ results in static horizons. However, when $N > 0$ quantum effects enable evaporation to occur and the horizon positions can be found to move. If one solves the equations of motion perturbativly, it can be shown that at first order, under the initial condition that the horizons are slightly peturbed (by σ_0), the separation of the horizons $\delta(t)$ becomes:

$$\delta(t) \approx \sigma_0\left(1 - \frac{1}{3}N\Lambda t^2\right)$$

This means the black hole increases back towards the maximal size once perturbed in this mode. It *Anti-Evaporates*! It turns out however, that this is not the only mode, and that the other modes lead to evaporation.

Conclusion

In conclusion, this project has explored key aspects of quantum field theory in curved spacetime, with a focus on black hole dynamics. Starting with the Unruh effect, we examined how accelerating observers perceive particle creation in a vacuum, leading to the understanding that particle and vacuum states are observer-dependent. Building on this foundation, we studied Hawking radiation, demonstrating how black holes emit radiation due to quantum effects near their event horizons. Finally, we investigated the concept of black hole anti-evaporation in the Schwarzschild-de Sitter spacetime, where quantum corrections can lead to unexpected dynamics, such as the black hole's expansion after perturbations. These results underscore that even in the absence of a theory of quantum gravity, many interesting aspects of black holes and horizons in general can be studied, paving the way for future advancements towards such a theory.

References

[1] Raphael Bousso and Stephen W Hawking. "(Anti-) evaporation of Schwarzschild–de Sitter black holes". In: *Physical Review D* 57.4 (1998), p. 2436.

[2] Stephen W Hawking. "Particle creation by black holes". In: *Communications in mathematical physics* 43.3 (1975), pp. 199–220.

[3] William G Unruh. "Notes on black-hole evaporation". In: *Physical Review D* 14.4 (1976), p. 870.