Dark Energy

E677D

May 2026

Abstract

This essay[1] examines Dark Energy (DE), the unknown mechanism driving accelerated expansion of our universe. While the cosmological constant Λ is the standard candidate, its theoretical fine tuning and mounting tensions with recent observations suggest our current understanding may be incomplete. We explore various modifications of gravity proposed to resolve these issues and present the Effective Field Theory of Dark Energy (EFTDE) as a robust, unifying framework for describing such models.

Acknowledgements

I would like to thank Enrico for his helpful comments and valuable discussions during the essay meetings and for setting such an interesting essay title. I would also like to thank my Family for their support throughout the year and all my friends in the part III room for the good company.

[1] In this essay we use the conventions $c = 1 = \hbar$ and the mostly plus metric $(-, +, +, +)$.

"Louise tells the story that every other day I would step out of the shower, and with a starry look report, "Honey, I think I have solved the cosmological constant!" The next day I would step out of the shower and muse, defeatedly, "No, ...""

-*Steven Weinberg* [1]

Contents

1 The Cosmological Constant

Our current best understanding of Dark Energy (DE) comes in the form of a Cosmological Constant Λ. This constant can be viewed as a simple addition to the construction of General Relativity (GR), which is best understood from the perspective of an action principle. When Einstein began to apply general relativity to the whole universe in 1917 [2], he realized that the addition of any matter fields leads to a unstable universe that tended to collapse in on itself under gravitational attraction. His solution was to "modify" the existing equations by adding a new free parameter Λ that essentially acts a constant force that counteracts this collapse.

More specifically, under this addition the total action takes the form:

$$S = S_{EH}[g_{\mu\nu}] + S_{\rm M} = \frac{M_{\rm pl}^2}{2}\int d^4x\sqrt{-g}\,[R - 2\Lambda] + S_{\rm M} \tag{1.1}$$

Which give rise to the modified Einstein equations under the variation of the action:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_{\rm pl}^2}T_{\mu\nu} \tag{1.2}$$

Where $T_{\mu\nu} := -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm M}}{\delta g^{\mu\nu}}$ is the Energy Momentum Tensor. By the late 1920's the experimental evidence for the expansion of the universe (and the discovery of static spacetimes without matter [3]) meant that the cosmological constant was abandoned. In the present day, improved observations of our universes expansions reveal it to be consistent with the re-addition of Einstein's Λ to our action (see section 2).

There are two, perhaps conflicting, perspectives on this observation. On one hand from the point of view of pure general relativity this addition is completely natural and in fact necessary to writing down the most general theory of gravity. In this perspective, Λ is nothing more then a sort of geometric property of general relativity and its value or size is no more mysterious then that of Newtons constant G. See [4] for example.

On the other hand, the addition of a Λ term to the action is extremely strange to a field such as particle physics. We know of no substances in the universe, no particle in the standard model that can cause an expansion like the one we see. So even if Λ is just another parameter of general relativity, one still expects there to exist an underlying, more fundamental reason for its existence and size. In this sense, the cosmological constant is an unsatisfying solution to dark energy as it offers no clear insights to the underlying physics.

1.1 The Cosmological Constant Problem

It may be argued that the primary issues with cosmological constant lie in its conflict with theory. The formulation of our most successful physical theory to date, the standard model of particle physics, has made it clear that our best understanding of nature on the smallest of scales is described by Quantum Field Theory (QFT). However, a conflict arises, as the vacuum in QFT, which we will see should to contribute to Λ, does not have a vanishing energy density. Instead it adds a value far larger then the one we observe today. This problem is aptly named *The Cosmological Constant Problem.*

1.1.1 Vacuum Energy: The Old Cosmological Constant Problem

The old cosmological constant problem can essentially be phrased as "Why is Λ zero?". In the present day a better phrasing would be "Why is Λ so small?". To see why this might be an issue, consider writing down the vacuum energy for some scalar field in QFT: (see Appendix A for motivation and exact expressions).

$$\langle\rho\rangle = \frac{1}{V}\langle 0| H |0\rangle = 2\pi\int_0^{\Lambda_{\mathrm{UV}}} dp p^2\sqrt{p^2+m^2} \simeq \frac{\pi}{2}\Lambda_{\mathrm{UV}}^4 + \mathcal{O}(m^2\Lambda_{\mathrm{UV}}^2) \tag{1.3}$$

Here Λ_{UV} is the Ultra Violet cut-off scale of the theory, and it has been assumed that $\Lambda_{\mathrm{UV}} >> m^2$. The problem then arises from the fact that by definition the energy momentum tensor associated with the energy density of the vacuum must be Lorentz invariant[2]. This means that the vacuum expectation value of the energy momentum tensor is:

$$\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$$

Where the constant of proportionality has been fixed using $T_{00} = \rho$. With this identification it is easy to see by looking back at the Einstein equations 1.2, that this contribution 1.3 acts exactly like a cosmological constant term, with contribution:

$$\Lambda_{\mathrm{vac}} \simeq \frac{\Lambda_{\mathrm{UV}}^4}{M_{\mathrm{pl}}^2}$$

To get a numerical estimate for this value one may be optimistic and claim that the theory being considered is valid all the way up until the Planck scale, in which case $\Lambda_{\mathrm{UV}} = M_{\mathrm{pl}} = 2.4\times 10^{18}$ GeV which gives a cosmological constant of value:

$$\Lambda_{\mathrm{vac}} \simeq 10^{36}\ \mathrm{GeV}^2 \tag{1.4}$$

But the observed value of the cosmological constant today is[3]:

$$\Lambda_{\mathrm{obs}} \simeq 10^{-84}\ \mathrm{GeV}^2 \tag{1.5}$$

This shows clearly that the discrepancy between theory and experiment when it comes to the cosmological constant is of the order of ~ 120 magnitudes. To be more conservative we could consider the cut off to be below the Planck scale, which would lower the value of Λ_{vac}. For example the standard model has been tested at the LHC up to $\Lambda_{\mathrm{UV}} \sim$ 1TeV which results a prediction of $\Lambda_{\mathrm{vac}} \simeq 10^{-25}$ GeV^2. Reducing the gap to ~ 60 orders of magnitude, a large improvement but no where near enough to resolve the problem.

1.1.2 An Effective Description of the Problem

The problem outlined above seems to hinge on the high energy details of the theory and one might imagine there to exist some high energy formulation that ends up removing these issues entirely. However, we will show, following [6], that there are issues that remain at low energies that this sort of solution cannot evade.

To examine this it should be noted that the previous construction was very "classical" in the sense that we started from a classical action and quantized it. In fact in doing so we have made the naive mistake of assuming the quantities in our action to be observables. Instead we can split them up into parameters to "run" with the energy scale (but remain finite) and those that cancel the large

[2] Lorentz invariance for tensors is quantified by invariance under Lorentz transformations of the form $(\Lambda^T)_\nu{}^\sigma\eta_{\sigma\rho}\Lambda^\rho{}_\mu = \eta_{\mu\nu}$. This is only satisfied by the Minkowski metric $\eta_{\mu\nu}$ and in curved spacetimes the equivalence principle can be used to note that the metric $g_{\mu\nu}$ is the only tensor that reduces to $\eta_{\mu\nu}$ locally.

[3] Calculated from the Planck cosmological parameters [5], using $\Omega_\Lambda = 0.679$, $H_0 = 67.66$ km/s/Mpc and $\Lambda = 3H_0^2\Omega_\Lambda$.

divergences in the theory. This allows use for example to "renormalize" the expression 1.3 by adding a potential (see Appendix A) $V_0 = V_R + V_{\Lambda_{\text{UV}}}$ with a part $V_{\Lambda_{\text{UV}}}$ that cancels all the dependence of Λ_{UV} in ρ_{vac} and a finite remaining part V_R. With this 1.3 becomes:

$$\langle\rho\rangle = V_R + c_R m_R^4 + \mathcal{O}(\frac{m^6}{\Lambda_{\text{UV}}^2})$$

Where all constants may also receive a (finite) renormalization, but should now be independent of Λ_{UV}. The above splitting happens at the cutoff scale, but below this the parameters of the action become dependent on the energy scale μ. The action for another scale $\mu' < \mu$ can be obtained in the path integral formulation by integrating out the higher energy modes of the field, which in turn changes the dependence of the parameters of the theory. This action is known as a *Wilson action* and it is in this sense that we can begin to think of our theory as an *Effective Field Theory* (EFT) .

In order to obtain a small vacuum energy one could choose the renormalization scheme such that any factors containing m_R^4 also get absorbed (as these are large compared to the observed value 1.5) leaving just the small constant that we observe. However, there is an ambiguity as to what energy scale this should be done at. In fact if we choose the energy scale μ to be just above the mass of the electron [4], then the energy density will contribute:

$$\Lambda = \frac{1}{M_{\text{PL}}^2}\langle\rho\rangle \simeq \frac{1}{M_{\text{PL}}^2}\left[V_0 + c_e m_e^4\right]$$

Then for μ below the mass of the electron (where we expect cosmology to be valid), the electron fields get integrated out (see Appendix A.1), leaving just:

$$\Lambda \simeq \frac{1}{M_{\text{PL}}^2} V_1$$

Where the potential V_0 has been renormalized. The problem then is that in order for Λ to remain small it is required that V_0 and $c_e m_e^4$ cancel to many decimal places as $m_e^4/M_{\text{PL}}^2 \simeq 10^{-50}\text{GeV}^2$. This is known as *Fine Tuning* and happens each time a heavy field is integrated out. From this point of view, it is very strange that we live in a universe, with a cosmological constant as small as 1.5.

1.2 The Anthropic Principle

Before moving on, it is worth noting a simple proposed solution to the issues of the cosmological constant problem, known as the *anthropic principle*. As put by Weinberg [3] this is the statement that "the world is the way it is, at least in part, because otherwise there would be no one to ask why it is the way it is". Weinberg was the primary proponent of this reasoning applied to the cosmological constant and can be quoted as saying "With my anthropic principle, I felt and still do feel that I had come to the rescue." [1].

The anthropic principle attempts to ease the issues we have raised previously about the interplay between the QFT and the cosmological constant by highlighting that it would be impossible for life to form in universes where Λ has the values set by the cut-off scale Λ_{UV} or the masses of particles. Instead, it must be the case, purely because we are here to see it, that these large contributions are cancelled by some other contribution, such that the total Λ is small. Weinberg gave this argument some numerical weight in 1987 by calculating an upper bound on how large Λ could be before it begins to forbid the

[4] We are ignoring any lighter fields such as the neutrinos here.

formation of gravitationally bound states [7]. This results in an upper bound that can be expressed in terms of fraction densities as:

$$\Omega_\Lambda \lesssim 550\Omega_\mathrm{m}$$

This bound is two orders of magnitude less than what is observed today, which is $\Omega_\Lambda \simeq 2.21\Omega_\mathrm{m}$. Later in 1997 Weinberg and others performed a more in depth calculation and found that a value of $\Omega_\Lambda \lesssim 3\Omega_\mathrm{m}$ was "a reasonably likely value to observe" [8]. Notably, this analysis was before the 1998 measurement of a non-zero Λ [9], making these bounds a sort of prediction.

The strength of any anthropic argument for the selection of the value of a parameter relies heavily on the existence of multiple realizations of that parameter[5]. When this type of reasoning is applied to the cosmological constant problem, it is difficult to consider the anthropic principle a serious solution as we simply lack evidence for any sort of multiple realization mechanism that would produce different values of Λ. Until such a mechanism is discovered it remains a better idea to search for alternative solutions to the cosmological constant problem.

1.3 The Coincidence Problem

One last issue that should be mentioned is the *Coincidence Problem*, which is the observation that at current times the fractional of density of dark energy Ω_Λ and matter Ω_m are of the same order of magnitude. Essentially, this is viewed as a problem as we expect cosmology to follow the Copernican principle[6] and hence view it strange that we live at a time when the universe is not dominated by one substance. For this reason the problem is also known as the "*Why now? problem*".

The problem is perhaps best illustrated by the left plot in Figure 1 adapted from [11], where it can be seen that in terms of redshift, the matter dark energy equality is very recent and has lasted for only a small fraction of the overall redshift of our universe. However, as was pointed out in [4] and followed up on by [11], this is only a matter of the choice of redshift as a co-ordinate. In the right plot of Figure 1 when the redshift z is replaced by cosmological time, the period after matter dark energy equality now makes up roughly one fourth of the total time since the big bang. Hence, with cosmological time as a co-ordinate our existence during this period looks far less extreme.

The real problem here is determining the correct co-ordinate for interpretation. This would be the co-ordinate where the probability of life existing is uniformly distributed so that it could be correctly determined whether the time of our existence is special. However, as soon as one starts asking such questions about the probability of life existing, the same anthropic principle we had above comes to dominate the discussion[7]. In the anthropic mindset, we know that we could not have existed far into the past (where matter dominated) as the universe needed time to form galaxies and stars with high metallicity. Similarly, we could not exist much later into the future (where dark energy will dominate) as the rate of star formation will decline and stars will begin to die, leaving insufficient conditions for life to form. In essence, the co-incidence problem is an anthropic problem, it is only natural that it should have an anthropic solution.

While the coincidence might suggest a coupling between the dark sectors or a need for modified gravity, the anthropic perspective suggests that this 'problem' may not necessitate a departure from Λ.

[5] For example, the anthropic principle can accurately explain why we happen to live on a planet in the habitable zone (assuming that liquid water is needed for life), as there are many planets at many different radii from the sun. If we lived in a universe with one planet that just happened to exist in the habitable zone, we might be more suspicious.

[6] This is the statement that humans are not privileged observers [10].

[7] It should be noted that there are no qualms with the usage of the anthropic principle here as the problem inherently includes the multiple realisations needed to make the argument. The different times at which we could live are the different realizations.

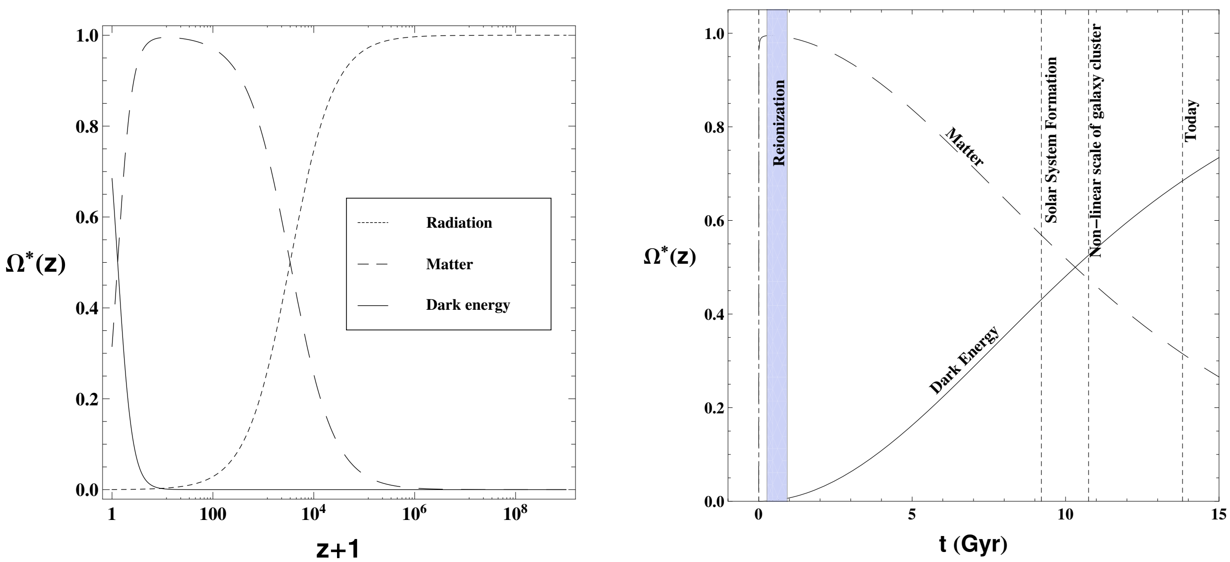

Figure 1: (Left) The fractional energy density (denoted here as Ω^*) as a function of redshift for radiation, matter and dark energy. (Right) Same fractional energy density but now as a function cosmological time (i.e. age of the universe). Both plots are adapted from [11].

2 Observational Probes

Having outlined the various problems associated with a cosmological constant we now turn to give a brief overview of the different observations that provide measurements and constraints of dark energy. We first focus on how type 1a supernovae were first used measure the universe's accelerated expansion and hence a non-zero value of Λ, before explaining how more recent observations of baryon acoustic oscillations and the Cosmic Microwave Background (CMB) allow us to measure the expansion history with remarkable precision.

2.1 Measurement of Expansion from Supernovae

Type 1a Supernovae (SNe) are the stellar explosions from white dwarfs that accrete mass above the Chandrasekhar limit of $\sim 1.44 M_\odot$. Since the white dwarfs have approximately the same mass (close to the Chandrasekhar mass) before the thermonuclear explosion, the resulting light curves of different type 1a supernovae are quite similar, making them ideal as "standard candles" for measuring distances in cosmology. Calibration of supernovae distances was completed in the 1990's by studying the light curves of supernovae in galaxies of known distances[8]. These light curves peak and then decline after the initial explosion, with a calculable relation between this decline time and the luminosity L as stars with lower luminosities are observed to dim quicker [12]. From observations of the flux F, the luminosity distance D_L can be inferred and related to Ω_Λ, through $H(z)$ (Assuming $\Omega_k = 0$) as:

$$D_L = \sqrt{\frac{L}{4\pi F}} = (1+z)\int_0^z \frac{dz'}{H(z')}$$

In 1998 two separate teams [9] and [13] used ~ 50 supernovae to show that a universe with $\Lambda = 0$ was statistically unfavoured based on the brightness of the observed supernovae as a function of redshift.

[8]These distances were known from the existence of Cepheid variable stars that have oscillations in their brightness, with a known relation between the oscillations and their overall luminosity.

This can be seen in Figure 2, where $m_B = 5\log_{10}(D_L) + \text{const}$ is the effective apparent magnitude.

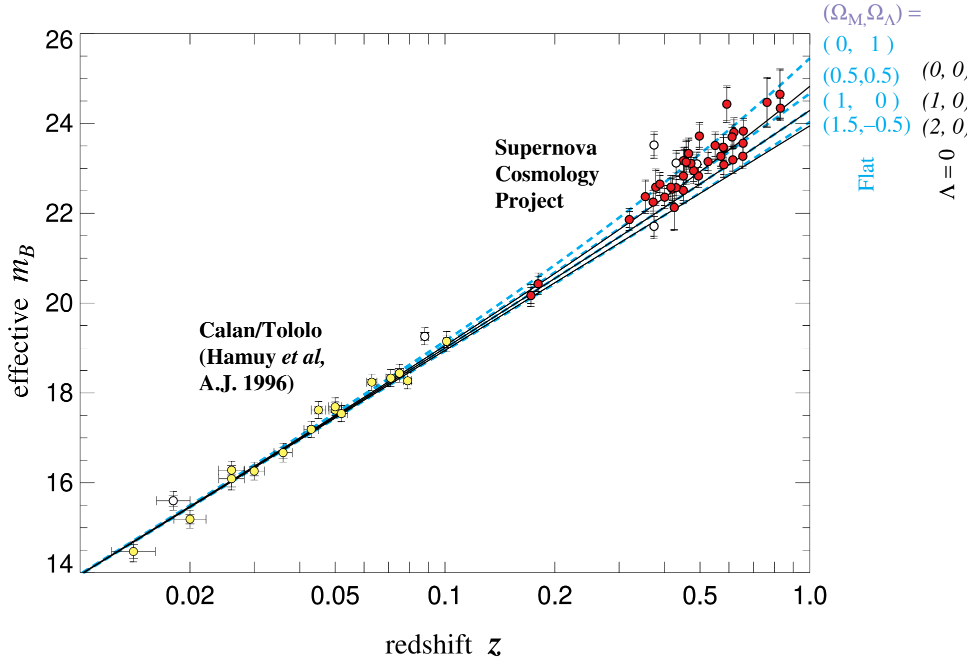

Figure 2: Effective Magnitude of 42 high (red) and 18 low (yellow) redshift supernovae as a function of redshift z. Solid and dashed lines show the different theoretical predictions of models with and without $\Lambda > 0$. Figure is a adapted from [13].

In the present day we have a far larger sample set of supernovae, thanks to the Dark Energy Survey (DES). Recent data analysis has involved over 1500 high redshift supernovae [14] and the newly operating Vera Rubin Observatory is set to see millions of supernovae over the next decade. This will vastly further the constraints of dark energy, the current status of which is reviewed in section 2.4.

2.2 Baryon Acoustic Oscillations

Baryon Acoustic Oscillations (BAO) are primordial sounds waves of matter in the early universe that can be observed now as small ripples in the galaxy power spectrum. They are a distinct prediction of the evolution of baryons and photons in the presence of a Cold Dark Mater (CDM) and are also imprinted in the CMB. The observations of BAO are interesting from the perspective of dark energy as the oscillation in Fourier space corresponds to a peak in position space that essentially acts as a cosmic "ruler". This means it is some length scale that we can measure all across the sky to get an idea of what the expansion history $H(z)$ has looked like and hence place constraints on dark energy.

To see where BAO come from it is first important to consider the far past at redshifts $z > 1100$, before recombination and the emission of the CMB photons. Here, the baryons and photons can be described as a single fluid[9] with overdensity δ_γ due to interactions from Thomson scattering. By solving the linearised Einstein equations (see chapter 6 and 7 of [15]) δ_γ can be shown to satisfy in, Fourier space, the equation of a damped harmonic oscillator:

$$\delta_\gamma'' + \frac{R_b \mathcal{H}}{1 + R_b}\delta_\gamma' + c_s^2 k^2 \delta_\gamma = 0$$

[9] This is known as the *Tight Coupling Approximation*.

Here, c_s^2 is the speed of sound, $'$ refers to a derivative wrt to conformal time η and $R_b := \frac{3}{4}\frac{\bar{\rho}_b}{\bar{\rho}_\gamma}$. Since $c_s = c_s(\eta)$, a solution can be found using the WKB approximation with the ansatz $\delta_\gamma = A(\eta)\exp\left[\pm ik\int_0^\eta c_s(\tilde{\eta})d\tilde{\eta}\right]$, resulting in [15]:

$$\delta_\gamma^{\text{fast}} = \frac{1}{(1+R_b)^{\frac{1}{4}}}\left[A\cos(kr_s) + B\sin(kr_s)\right], \quad r_s(\eta) := \int_0^\eta c_s(\tilde{\eta})d\tilde{\eta} \tag{2.1}$$

Where $r_s(\eta)$ is the (co-moving) *sound horizon* and can be interpreted as the the distance travelled by sound waves until the time of decoupling z_d. Here, $\cos(kr_s) = T_b^{\text{rec}}(\mathbf{k}, \eta_{\text{rec}})$ can be recognised as the transfer function that transports the initial conditions from the end of inflation to recombination. After decoupling, these sound waves are frozen, with different frequencies coming to a halt at different amplitudes, creating an oscillating effect. Finally, solving the linearised Einstein equations during matter domination we can evolve these frozen perturbations from recombination to the present day, with the result being the *Mezaros Equation* [16], which has a growing solution of $\delta_m \propto a(\eta)$. This means splitting the matter into fractions of baryonic and dark matter[10] $\delta_m = f_b\delta_b + f_c\delta_c$, allows us to write:

$$\begin{aligned}\delta_m(\mathbf{k},\eta) &= \left[f_b\delta_b(\mathbf{k},\eta_{\text{rec}}) + f_c\delta_c(\mathbf{k},\eta_{\text{rec}})\right]\frac{a(\eta)}{a(\eta_{\text{rec}})} \\ &= \left(1 + \frac{f_b T_b^{\text{rec}}(\mathbf{k})}{f_c T_c^{\text{rec}}(\mathbf{k})}\right) f_c\delta_c(\mathbf{k},\eta)\end{aligned}$$

Since matter is largely composed of dark matter, this second term in the brackets is sub-leading. $T_b^{\text{rec}} \propto \cos(kr_s)$ is the oscillating solution found previously, and since dark matter does not oscillate in this way, the result is small wiggles on top of the matter power spectrum in Fourier space. These oscillations along with the corresponding position space bump are shown in Figure 3.

2.2.1 BAO Constraints on Dark Energy

BAO can be used to probe our expansion history through the measurement of two key length scales both of which can be compared to the known value of the sound horizon $r_d = r_s(z_d)$ 2.1. First, by observing the statistics of galaxies at similar red shift z but different angular separation, allows the determination of the co-moving distance[11] to these galaxies:

$$D_M(z) = \int_0^z \frac{dz}{H(z)} \tag{2.2}$$

This is depicted in the left of Figure 4. The second distance scale can be measured by analysing the statistics of galaxies along the line of sight, as is shown on the right of Figure 4. The light we see will have come from galaxies separated by a red shift Δz hence a small amount of expansion will have take place between these times. This when compared to the standard ruler r_d allows us to determine $H(z)$. Specifically, the change in the co-moving distance ΔD_m over this small redshift change Δz is by 2.2:

$$D_H(z) = \frac{1}{H(z)} = \frac{\Delta D_M}{\Delta z} \tag{2.3}$$

The Dark Energy Spectroscopic Instrument (DESI) BAO data comes from seven sample collections of galaxies at different redshifts. The measurements of $D_M(z)$ 2.2 and $D_H(z)$ 2.3, allow the fitting of

[10] These fractions f_b, f_c are constant in η as the overdensities satisfy the same evolution equation.

[11] This is the distance traveled by light from the galaxies to us now.

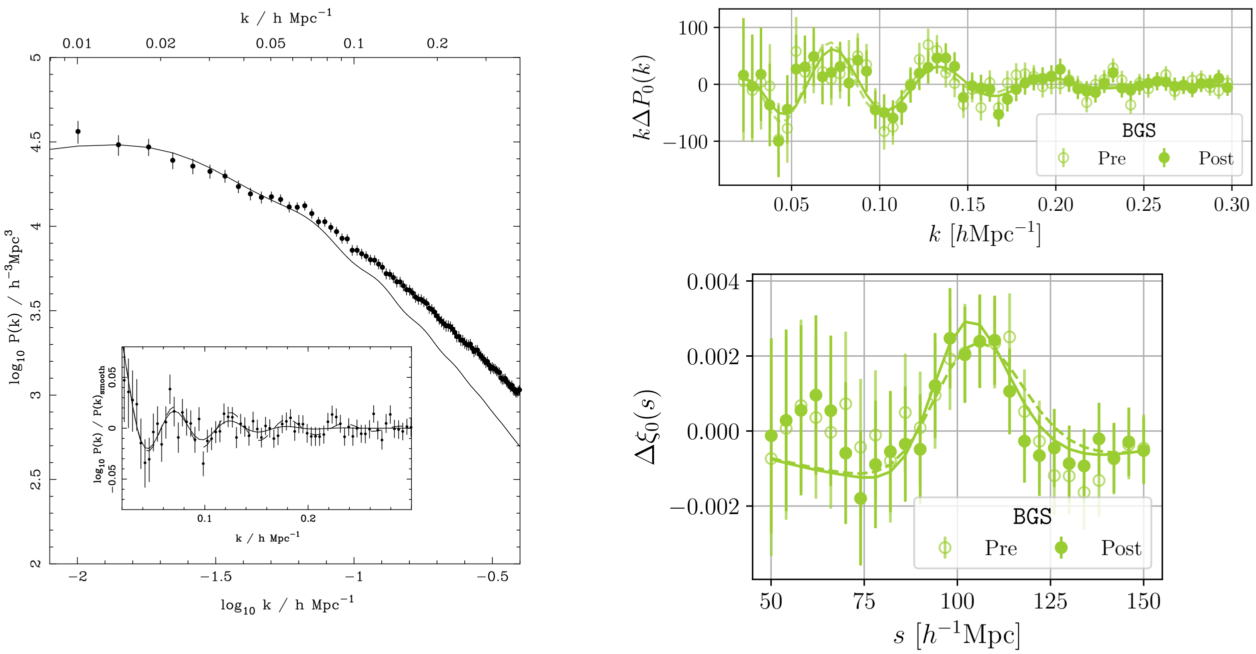

Figure 3: (Left) Matter power spectrum from Sloan Digital Sky Survey 2006, solid curve shows linear perturbation theory expectation and the inset shows residual BAO oscillations. Figure is adapted from [17]. (Right) Power spectrum residuals from recent DESI Bright Galaxy Survey 2024. Top shows momentum space oscillations, bottom shows position space bump at $r_s \simeq 150Mpc$. Adapted from [18].

$H(z)$ at these redshifts, which can be compared to the ΛCDM prediction, as seen in the left panel of Figure 5. So far, the DESI Collaboration has measured the spectra of almost 30 million galaxies [18], making the statistics of the BAO oscillations quite constraining. In section 2.4 we review the current bounds on dark energy in light of the BAO data.

2.3 The Cosmic Microwave Background

The Cosmic Microwave Background (CMB) provides, by far, the most constraining data for our cosmological models, by fitting the 6 parameters of ΛCDM to its power spectrum. The CMB measures dark energy in a very similar way to BAO. The CMB's own acoustic oscillations set a sharply defined angular scale θ_* that can be used to determine D_M 2.2 as:

$$\theta_* = \frac{r_d}{D_M}$$

However, CMB data alone can struggle to constrain how dark energy modifications can differ from Λ, purely due to the fact that the CMB was released long before dark energy started to dominate. The result of this is that the CMB alone cannot rule out vast areas of the parameter space; the expansion history between the CMB and now can always be chosen in such a ways as to make the CMB look the same[12]. Nevertheless, in combination with data from other experiments, the CMB's precision measurements of the other ΛCDM parameters make it an excellent complementary probe.

[12] See section 7.4 of [5] for further details.

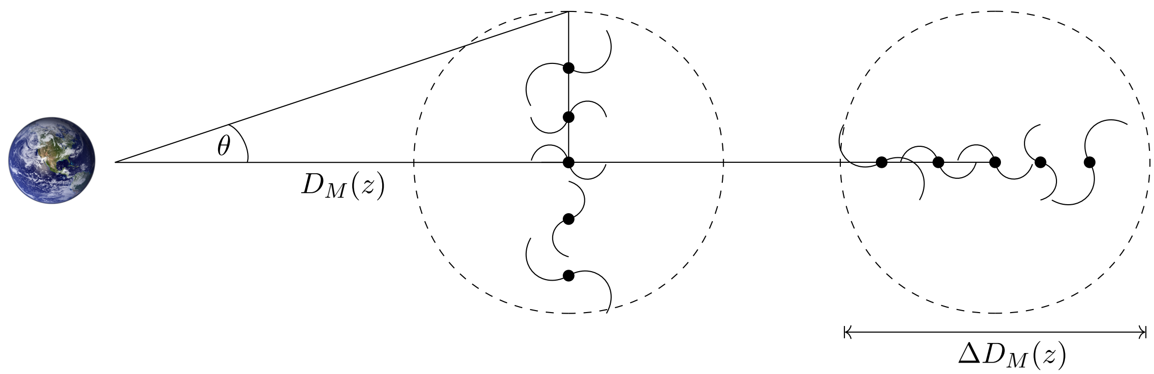

Figure 4: Schematic of how the statistics of galaxies and BAO can measure two length scales. Left shows angular measurements, which can determine the co-moving distance $D_M(z)$. Right shows measurments of galaxies at slightly different redshifts Δz, which measures the expansion rate.

2.4 The Case for Dynamical Dark Energy

2.4.1 Dark Energy Equation of State

By shifting what side of the Einstein equations terms appear on, one can always describe a term in the equations of motion as arising from some stress energy tensor $T_{\mu\nu}$. In this approach one can take the $\Lambda g_{\mu\nu}$ term in 1.2 and describe it as a fluid that permeates all of space. A convenient way to characterize fluids is by their *Equation of State* (EoS) ω, which is defined as the ratio of pressure over volume $\omega := P/\rho$. If one makes the perfect fluid approximation[13] then the conservation law $\nabla_\mu T^{\mu\nu} = 0$ can be written as:

$$\frac{\dot{\rho}}{\rho} = -3H(1 + \omega(z))$$

Where the equation of state $\omega(z)$ may depend on redshift. For $\Lambda = \text{constant} \implies \rho = \text{constant}$ and hence $\omega = -1$ always. However, it would be interesting to see if models that differ slightly from $\omega = -1$ over time, may fit current observational data better. In order to determine whether this is the case, it is necessary to parametrize the equation of state in some manner. Originally, parametrizations such as $\omega(z) = \omega_0 + \omega_1 z$ were proposed as a sort of Taylor expansion, though this is not useful for redshifts $1 < z < 2$, where in fact a lot of BAO and supernovae data comes from. It was then proposed by [19] and [20] that the following parametrization[14], known as the CPL parametrization, should be used instead:

$$\omega(z) = \omega_0 + \omega_a(1 - a) = \omega_0 + \omega_a \frac{z}{1+z} \tag{2.4}$$

It can be seen that ΛCDM is the unique set of values $\omega_0 = -1,\ \omega_a = 0$.

2.4.2 Observational Evidence

Over the past few years observations have become precise enough to fit ω_0 and ω_a to the data. Most interestingly, recent BAO measurements by DESI have shown, in combination with other data sets

[13] This is essentially neglecting any anisotropic stresses, a good approximation for the evolution of the background.

[14] It is worth pointing out, as is highlighted in [21] that this parametrization is not a Taylor expansion near the present, it is simply a method of parametrisation that should allow for the distinction between a constant vs dynamical EoS, specifically for observations between $1 < z < 2$ as was the motivation for its introduction in [19].

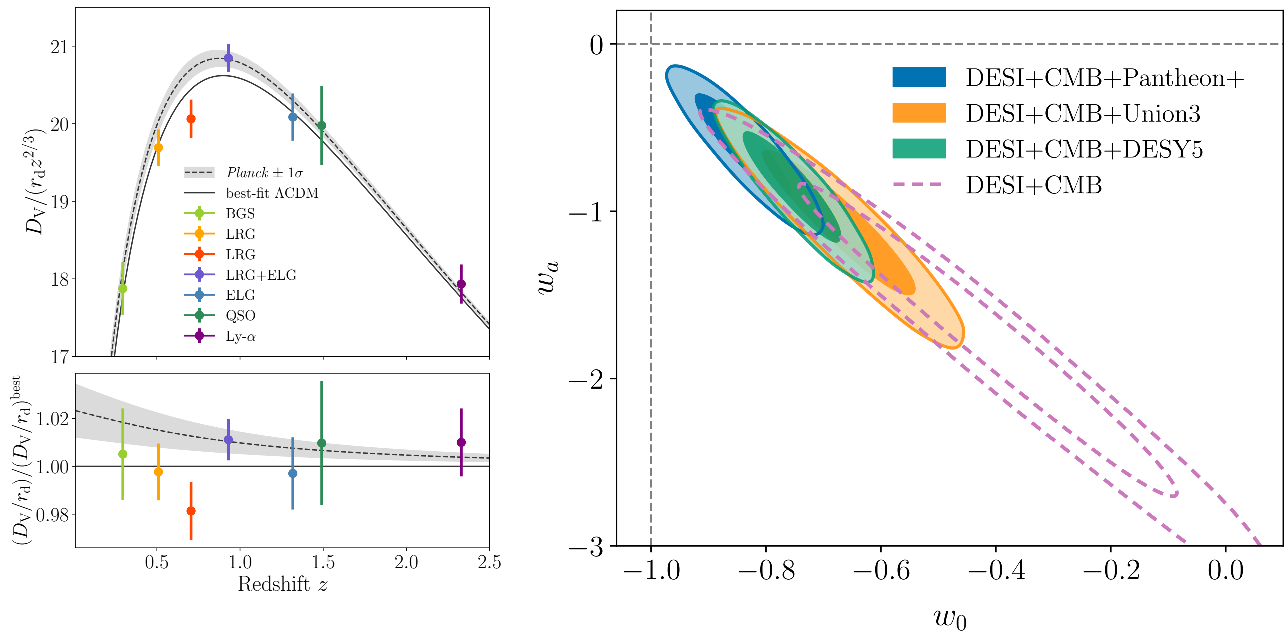

Figure 5: (Left) DESI BAO distance scales at different redshits using the ratio of the angle averaged distance $D_V := (zD_M^2 D_H)$ to the sound horizon (factor of $z^{3/2}$ is for plotting convenience). Lower shows ΛCDM cannot fit BAO and CMB at the same time. Adapted from [22]. (Right) 68% and 95% contours for ω_0 and ω_a as fitted to DESI+CMB and three different SNe catalogues. Adapted from [18]

(Supernovae and CMB), that their is a weak preference in the data for dynamical dark energy, when using the CPL parametrisation 2.4. An example of the 1σ and 2σ contour intervals are shown for the combination of DESI BAO + CMB data with 3 different supernovae catalogues is shown in right panel of Figure 5. In 2024, analysis of the DESI DR2 data [22] and a 2025 separate reanalysis [23] concluded that with the DESY5 supernovae catalogue $\omega_0\omega_a$CDM is favoured over ΛCDM at $(3.9 - 4.2)\sigma$. More recent reanalysis with improved calibrations of the supernovae data [24] has reduced this to 3.2σ. As it stands there is enough evidence to say that dynamical dark energy ($\omega_0\omega_a$CDM) is weakly favoured over ΛCDM, though it is still along way off a genuine 5σ discovery.

With future observations, such as by the Vera Rubin and Euclid telescopes, along with further data releases from DESI, it is quite possible that there will be enough data to resolve whether this is a genuine tension or not in the near future. With the latest tension being a reduction from $4.2\sigma \to 3.2\sigma$, there are certainly questions as to whether this preference for $\omega_0\omega_a$CDM will disappear entirely upon some reanalysis. It is worth pointing out, that while no one tension has been shown to be strong, there is a slight preference across nearly all current data sets for $\omega_0\omega_a$CDM. This is best illustrated by the summary of the different data combinations in Figure 6[15] and further in Figure 1 of [25].

[15] It is worth noting, that since Figure 6 is a 1-d plot, it does not capture the 2d distribution of ω_0 and ω_a seen in Figure 5. For example on the left, the point $\omega_0 = -1$ also includes $\omega_a \neq 0$.

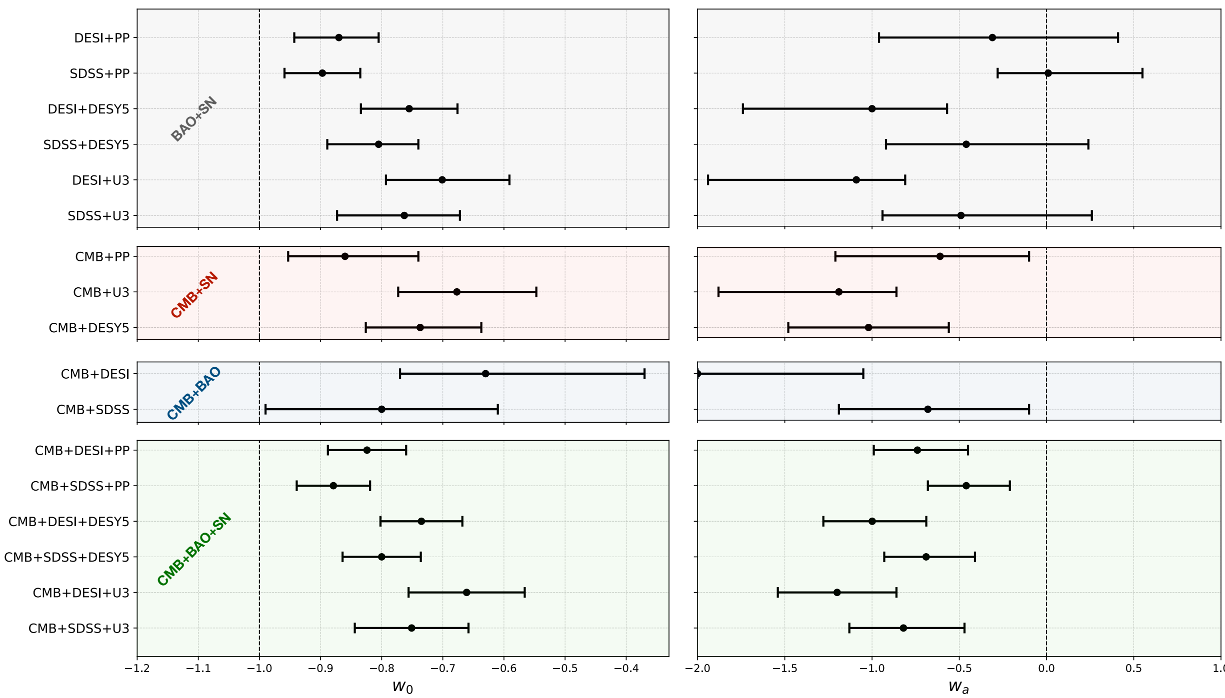

Figure 6: Whisker plot of 68% confidence levels for ω_0 and ω_a parameters for variety of cosmological probes. Adapted from [25].

3 Dynamical Models and Modified Gravity

As the preceding sections indicate, modelling dark energy as a cosmological constant Λ faces inherent theoretical dissatisfaction and growing observational tensions. This motivates the exciting search for alternative explanations of dark energy that would seek ease some of these issues. In this section we will review some of the theoretical constraints and issues facing the introduction of such models before giving a brief overview of some of the different approaches that can be taken in section 3.4.

3.1 Weinberg's No-Go Theorem

In section 1.1.1 and 1.1.2 it was outlined how it is expected that QFT should contribute a large vacuum energy to the cosmological constant Λ. A natural solution to this problem is to have Λ be a dynamical variable that is by some mechanism, destined to tend towards the value $\Lambda = 0$[16]. If a mechanism like this was found it would completely remove the any of these fine tuning issues. However, as was shown by Weinberg in 1989 [3] it is impossible to construct such theories with in general relativity, without them inherently involving fine tuning in the construction. A detailed sketch of the proof can be found in Appendix A of [26].

No go theorems such as this one are important not just for their results, but also for the list of assumptions that are made to complete the proof, as these can be used as a starting point for evading the theorem's consequences[17]. From this point of view it it worth noting the assumptions that have gone into this theorem and what could be done to break them:

- Locality $\rightarrow$ Introduce non-local terms.

[16] For example it may be controlled by some scalar field that has a equilibrium position where $\Lambda = 0$.

[17] A famous example would be how both supersymmetry and conformal symmetry evade the Coleman Mandula theorem.

- Constant fields → Instead allow some spatial dependence.
- Poincaré Invariance → Introduce models that break these symmetries.
- Diffeomorphism Invariance → Modify the postulates of general relativity to break this.

If one takes the fine tuning issue of the cosmological constant seriously, then these are the starting points for model building.

3.2 The Fifth Force

A remarkable fact about general relativity is that, if one requires Lorentz invariance[18], then the weak field limit of GR reveals itself as the unique interacting theory of a massless spin-2 particle[19]. In this picture, perturbations of the metric $h_{\mu\nu}$ around flat space $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$ correspond to the graviton and the coupling of the vacuum energy to gravity, that was discussed in section 1.1.1 and appendix A.1, can be visualized as the sum of external gravitons attached to the vacuum loops.

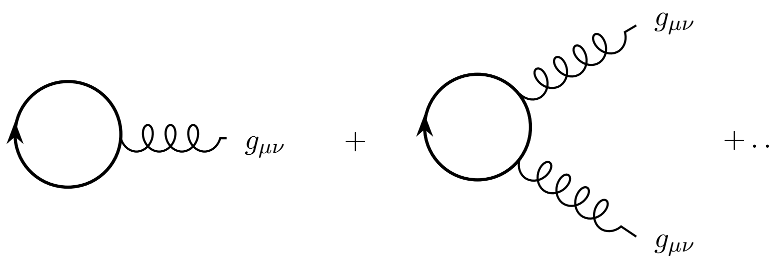

Figure 7: Diagrammatic contribution of the vacuum energy of the standard model fields (solid black line) to the cosmological constant via coupling to external gravitons. Recreation of Figure 1 in [28].

The uniqueness of GR as a theory of a massless spin-2 particle, means that new degrees of freedom (dof) are needed if one wants to change anything about how the vacuum couples to gravity. However, introducing any such dof will lead to new predictions, even at energy scales we know very well. To see how this problem manifests itself formally consider adding a scalar degree of freedom ϕ that couples to the standard model fields though the following diagram:

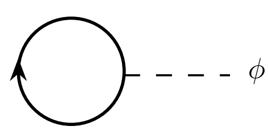

Figure 8: Tadpole

If we have some standard model field ψ, then Figure 8 tells us that the correlation function $\langle \phi(x_1)\psi(x_2)\psi(x_3)\rangle \neq 0$. But then, by *Unitarity*, it must be the case that ϕ is included in the completeness relations $\mathbb{1} = \sum_X \int d\Pi_X |X\rangle\langle X|$. Hence, it can be shown that greens functions will have a pole when a sum of the subset of the momentum add up to p, where $p^2 = -m_\phi^2$. See section 24.4 of [29] for details[20]. In momentum space this can be written as [29]:

$$G(p_1,\ldots,p_n) = (2\pi)^4\delta^{(4)}\left(\sum p_i\right)\frac{i}{p^2+m_\phi^2+i\varepsilon}\mathcal{M}_{1,r}\mathcal{M}^\dagger_{r+1,n} + \text{extra}$$

[18] Another requirement of Lorentz invariance is that coupling of the graviton to every particle is the same [27]. In other words, gravity is universal. This result makes it clear why diagrams such as those in Figure 7 are expected exist.

[19] See Appendix D of [26] for a self contained proof.

[20] This proof is a completely non-perturbative and has no reliance on Feynman diagrams or perturbation theory.

Where $p := p_1 + \ldots + p_r$. If we consider the case $n = 4$ and $r = 2$, this result essentially tells that the following diagram must exist:

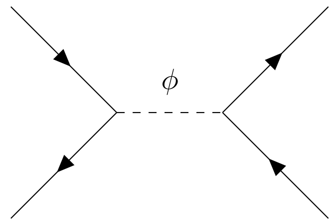

Figure 9: Exchange

So unitarity implies this scalar mediates a new "fifth" force between the particles of the standard model.

3.3 Screening Mechanisms

The most natural explanation for why we do not observe any fifth forces on Solar System scales is that something about the scalar ϕ is environment dependent. This type of effect is known as *Screening*. It may seem ad hoc that this type of mechanism would exist with out some very specific requirements, but we show in Appendix B that consideration of a very general scalar conformally coupled to matter can have the degrees of freedom to give rise to such an effect. In particular, we show that the force mediated by ϕ between two standard model particles, as in the diagram 9 is given by:

$$V(r) \propto \frac{g'(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})} \frac{1}{r} e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})}c_s(\bar{\phi})} r}$$

This makes it clear how the value of background $\bar{\phi}$ can change how observable the force is. There are then a number of known mechanisms that can link the background values of the field to the matter density[21], or the depth of the gravitational well. These include:

- **The Chameleon Mechanism**: Here, a scalar field develops a density dependant mass, usually due to a specific potential.[22]
- **The Symmetron Mechanism**: Here, a scalar field is coupled to matter through a Vacuum Expectation Value (VEV) such that the coupling vanishes when the symmetry is unbroken.
- **The Vainshtein Effect**: Here, higher derivative terms are added in such a way as the avoid ghosts and the scalar depends on the local density.

While no distinct model has distinguished itself as a clear candidate for dark energy, these mechanisms are promising indications that there could exist such models that have evaded our detection so far.

3.4 Overview of Different Models

Here we give a brief overview of a couple of different approaches to modifying or adding extra degrees of freedom to the gravitational sector. This is by no means an exhaustive list, for that we refer the reader to [31] and [32].

Quintessence and k-Essence

Perhaps the simplest extension of the degrees of freedom involved in the gravitational sector is to simply add a canonical scalar field with a potential to the action that is minimally coupled to gravity. Such

[21]The simplest solution is that the couplings are inherently small, though then it is difficult to avoid fine tuning.

[22]In particular recent work has shown that a the coupling of scalar conformally to the Standard model leads to the scalar obtaining a background energy density dependence [30].

models go under the name of *Quintessence*[23], The quintessence action most generally takes the form:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - V(\phi)\right] \tag{3.1}$$

If one models the scalar field as a perfect fluid and $\phi = \phi(t)$ then simple expressions for the pressure P and energy density ρ can be obtained, resulting in:

$$\omega = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi} - V}{\frac{1}{2}\dot{\phi} + V}$$

This expression makes it clear how quintessence can lead to a dynamical equation of state. The main theoretical issues facing quintessence are fine tuning due to Weinberg's no go theorem 3.1.

A simple generalisation of this model is to also allow arbitrary functions of the kinetic scalar $X := -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$. Note that this still only contains first derivatives. This model is known as *k-essence* and the action is simply written in terms of some arbitrary function P:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}R + P(\phi, X)\right] \tag{3.2}$$

Scalar Tensor Theories

The next step up from quintessence is to consider a theory of gravity governed by a scalar and a tensor, coupled in non-minimal way. The action is generally written as[24]:

$$S = \int d^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\partial\phi)^2 - V(\phi)\right] + S_m[g_{\mu\nu}, \psi_i] \tag{3.3}$$

As discussed in Appendix C, the metric can be redefined so that $\phi R \to R$, but this will result in regular matter no longer following geodesics. The most studied of these theories is *Brans-Dicke Theory*, which has $\omega =$ constant and $V = 0$. In this case the field equations are well understood and can be shown to lead to a number of effects such as a Newtons constant G which varies in timea.

Horndeski's Theory

To be even more general one can consider adding higher derivative terms of the scalar to the action. However, when one does so they need to be careful not to introduce Ostrogradski instabilities[25]. This motivated *Horndeski theory*, which is the most general scalar tensor theory of gravity for which all equations of motion contain no higher than second derivatives (and hence does not suffer from instabilities). The action [32], contains four arbitrary functions $G_i(\phi, X)$ of the field ϕ and X:

$$S = \int d^4x\sqrt{-g}\left[\sum_{i=2}^{5}\mathcal{L}_i\left[g_{\mu\nu}, \phi\right]\right] + S_m[g_{\mu\nu}, \psi_i], \tag{3.4}$$

$$\mathcal{L}_2 = G_2(\phi, X), \qquad \mathcal{L}_3 = G_3(\phi, X)\Box\phi, \qquad \mathcal{L}_4 = G_4(\phi, X)R + \partial_X G_4(\phi, X)\left[(\Box\phi)^2 - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\right],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\partial_X G_4(\phi, X)\left[(\Box\phi)^3 + 2\nabla_\mu\nabla^\nu\phi\nabla_\nu\nabla^\alpha\phi\nabla^\mu\nabla_\alpha\phi - 3\nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi\Box\phi\right]$$

[23] Owing to the fact that they cause a fifth force as discussed in section 3.2.

[24] More generally the first term could be $f(\phi)R$, but a field re-definition can always be performed to get it into this form.

[25] Adding higher derivatives leads to a Hamiltonian that is unbounded from below, see Appendix C of [26].

The derivative interactions means the theory can make use of Vainshtein screening to avoid producing a fifth force. Horndeski's theory has been used in the context of dark energy to probe self-tuning models that evade Weinberg's no go Theorem 3.1 by breaking Poincaré symmetry ??. In more recent times, further models (GLPV and subsequently DHOST) have found ways of expanding further upon this action, see [32].

4 The Effective Field Theory of Dark Energy

The wealth of models of dark energy and modify gravity, paired with the knowledge of what is actually measurable in cosmology, has motivated, over the past two decades, the usage of an *Effective Field Theory* (EFT) approach. The EFT approach enables a unifying perspective by encapsulating all models that have an extra scalar degree of freedom, while also extracting only the parts that are relevant to observations. The use of EFT in describing dark energy is heavily motivated by its usage in describing inflation and hence a lot of the formalism is borrowed from [33]. The main idea is to consider linear perturbations around a homogenous and isotropic FLRW background, with the EFT operators[26] A also split into background $\bar{A}$ and perturbations δA, with $A(t, \mathbf{x}) = \bar{A}(t) + \delta A(t, \mathbf{x})$[27].

4.1 The Unitary Gauge

In a typical EFT approaches involving a scalar field, such as Weinberg's EFT for inflation [34], one would expand in terms of derivatives and functions of ϕ, neglecting terms after a certain order . However, the higher order terms still contribute to the evolution of the background and hence, analysis has to be redone at each order that is considered. In what is known as the *Unitary Gauge*, this issue can be circumvented by using diffeomorphism invariance to remove perturbations of the field ϕ all together. To see this, consider the extra scalar degree of freedom ϕ expanded in terms of its background and perturbation:

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \delta\phi(t, \mathbf{x})$$

Diffeomorphism invariance means we can always change co-ordinates $x^\mu \to \tilde{x}^\mu = x^\mu + \epsilon^\mu(x)$. The invariance [28] $\tilde{\phi}(\tilde{x}) = \phi(x)$ then requires to linear order that:

$$\delta\tilde{\phi}(x) = \tilde{\phi}(x) - \bar{\tilde{\phi}}(x) = \phi(x) - \epsilon^\mu \partial_\mu \phi(x) - \bar{\phi}(x) = \delta\phi - \epsilon^\mu \partial_\mu \phi$$

To make $\delta\tilde{\phi} = 0$ we can choose[29] $\epsilon^\mu = (\epsilon^0, \vec{0})$ such that $\delta\phi = \epsilon^0 \dot{\bar{\phi}}$. The consequence of such a choice is that time diffeomorphisms are broken, t is now a function of the background field $\bar{\phi}$ meaning we have essentially chosen our time slices to coincide with constant $\phi = \text{const}$ hypersurfaces Σ_t.

In this gauge we can split all operators into parts in Σ_t and parts orthogonal, i.e. proportional to the normal vector n^μ. In the unitary gauge, since $\phi = \bar{\phi}(t)$, this normal vector will coincide with the velocity vector of the scalar field and hence the unitary gauge is often known as the *Velocity Orthonormal Gauge*:

$$u_\mu = -\frac{\partial_\mu \phi}{\sqrt{-\partial_\mu \phi \partial^\mu \phi}} \xrightarrow[\text{gauge}]{\text{unitary}} -\frac{\delta^0_\mu}{\sqrt{-g^{00}}} =: n_\mu \tag{4.1}$$

All that remains is to find operators that exist purely on the hypersurfaces Σ_t.

[26] Operators here refer to terms that can be written in the EFT action, eg the Ricci scalar R.

[27] The background operators will only be functions of time in order to preserve homogeneity.

[28] The choice is made that the background quantities do not change under such transformations and instead the changes are ascribed to the perturbation.

[29] Realistically, this is the only choice, breaking spatial diffeomorphisms would contradict homogeneity.

4.2 Action Ingredients

Since effective field theories seek to capture the low energy behaviour of theories defined at a higher scale, it is paramount for us to write down every possible operator that is allowed by symmetries. These operators will be constructed from the following building blocks:

- **Co-ordinate time** t: In the unitary gauge this preferred choice of *"clock" time* co-ordinate t, is free to appear in the action now that time diffeomorphisms are broken.

- **Normal Vector** n^μ: Which by its above definition 4.1 is $n^\mu \propto \delta_0^\nu$, meaning it simply picks out the time component of what ever it is contracted with.

- **Induced Spatial Metric** $h_{\mu\nu}$: Simply put, this is the part of the full metric $g_{\mu\nu}$ that "lives" in the hyper surfaces[30] and is defined as:

$$h_{\mu\nu} := g_{\mu\nu} + n_\mu n_\nu$$

  The induced metric acts as a projection operator onto the t = const hypersurfaces, as it can be seen that $h^\mu{}_\nu n^\nu = 0$ (and $h^\mu{}_\nu h^\nu_\sigma = h^\mu{}_\sigma$). If we are using the metric $g^{\mu\nu}$ and the normal vector n^μ, then it is redundant to use $h^{\mu\nu}$ itself, but it is still useful as a projection tool.

- **Extrinsic Curvature** $K_{\mu\nu}$: This describes how hypersurfaces are extrinsically curved, that is, how they deform in time as space expands[31]. This information is captured by how the normal vector of Σ_t changes. This leads naturally to the definition of the *Extrinsic Curvature*:

$$K_{\mu\nu} := h^\alpha{}_\mu h^\beta{}_\nu \nabla_\alpha n_\beta = h^\alpha{}_\mu \nabla_\alpha n_\nu \tag{4.2}$$

  As is clear from the appearance of the induced spatial metric in the definition of $K_{\mu\nu}$, this quantity also lives in the hypersurfaces Σ_t, and satisfies $K_{\mu\nu} n^\mu = 0 = K_{\mu\nu} n^\nu$.

- **Riemann tensor** $R_{\mu\nu\rho\sigma}$: Is the only tensor that can be constructed from the metric and its first and second derivatives and is linear in the second derivatives [36]. The induced Riemann tensor ${}^{(3)}R_{\alpha\beta\gamma\delta} = h^\mu{}_\alpha h^\nu{}_\beta h^\rho{}_\gamma h^\sigma{}_\delta R_{\mu\nu\rho\sigma}$ does not need to be considered, as on Σ_t, $R_{\mu\nu\rho\sigma}$, ${}^{(3)}R_{\alpha\beta\gamma\delta}$ and the $K_{\mu\nu}$ are related through the *Gauss-Codacci relations* [35], so it is redundant to use all three of these quantities. For convince, the convention is to use $R_{\mu\nu\rho\sigma}$ and $K_{\mu\nu}$.

4.2.1 EFT functions

Since breaking of temporal diffeomorphisms leads to the appearance of t as a parameter in the theory, it is consistent with the symmetries to write down an arbitrary function of t multiplying each of the operators. These are known as the *EFT Functions*. To see how these may come about from co-variant terms, consider some operator $A^{\mu\nu}$, and the scalar $A^{\mu\nu} n_\mu n_\nu$, which using 4.1 can be expressed as:

$$A^{\mu\nu} n_\mu n_\nu \xrightarrow[\text{gauge}]{\text{unitary}} -\frac{1}{g^{00}} A^{00} = \left[1 + \delta g^{00} + (\delta g^{00})^2 + \mathcal{O}\left((\delta g^{00})^3\right)\right]\left[\bar{A}^{00} + \delta A^{00}\right]$$

Where we have used the fact that $g^{00} = -1 + \delta g^{00}$. Recalling that every background operator is only a function of time $\bar{A}^{\mu\nu} = \bar{A}^{\mu\nu}(t)$, allows the perturbations to pick up functions of time:

$$A^{\mu\nu} n_\mu n_\nu \xrightarrow[\text{gauge}]{\text{unitary}} 2A^{00}(t) + A^{00}(t) g^{00} + \delta A^{00} + A^{00}(t)(\delta g^{00})^2 + \delta g^{00} \delta A^{00} + \mathcal{O}\left(\delta^3\right) \tag{4.3}$$

In this sort of decomposition the first two terms affect the background and the rest are perturbations.

[30] Often referred to as the *First Fundamental Form*.

[31] Extrinsic curvature can also be thought of as the bending of the hypersurfaces into different times (see figure 10.3 of [35]), but since Σ_ta has $t = const$ this aspect is not captured here.

4.2.2 The Action

We are now ready to write down the action, As we have explained in the above, significant simplifications come about when we use the fact that $n_\mu \propto \delta^0_\mu$ and expand in powers of the perturbations. Since linear perturbation theory is a good approximation in cosmology, we only need to expand the action up to quadratic perturbations (as these lead to linear EoM). The result is the following:

$$\begin{aligned} S =& S_m[g_{\mu\nu}, \psi_i] + \frac{1}{2}\int d^4x\sqrt{-g}\,\Big[M_{\rm pl}^2 f(t)R - 2\Lambda(t) - 2c(t)g^{00} \\ &+ M_2^4\left(\delta g^{00}\right)^2 - \bar{m}_1^3(t)\delta g^{00}\delta K - \bar{M}_2^2(t)\delta K^2 - \bar{M}_3^2(t)\delta K^\mu{}_\nu \delta K^\nu{}_\mu + \mu_1(t)\delta g^{00}\delta R + m_2^2 h^{\mu\nu}\partial_\mu \delta g^{00}\partial_\nu \delta g^{00} \\ &+ \lambda_1 \delta R^2 + \lambda_2 \delta R_{\mu\nu}\delta R^{\mu\nu} + \mu_1^2\delta g^{00}\delta R + \gamma_1 C^{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} + \gamma_2 \epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda} + \mathcal{O}(\delta^3)\Big] \end{aligned} \tag{4.4}$$

Here $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, see [37] for an expression. This action is organised such that terms containing background and linear perturbations only appear in the first line, the remaining two lines contain quadratic perturbations. The linear perturbations are hidden in the $R = \bar{R} + \delta R$ and $g^{00} = -1 + \delta g^{00}$ terms, and their coefficients are constrained by the background EoM.

4.3 The Stückelberg Trick

In using the unitary gauge the one extra degree of freedom from the scalar perturbation $\delta\phi$, seems to be lost. What has in fact happened is that by breaking time diffeomorphisms the metric has less constraints on it and hence inherits this degree of freedom. In the language of Spontaneous Symmetry Breaking (SSB) one can say that this extra degree of freedom is "eaten" by the metric (or the graviton). Once a gauge such as the unitary gauge has been chosen it is often useful to have a way to "unfix" and restore full diffeomorphism invariance. In the context of the unitary gauge, this mechanism is known as the *Stückelberg Trick*, and is achieved in this case by a further redefinition of the time co-ordinate:

$$t \to t + \pi(x), \quad \mathbf{x} \to \mathbf{x}$$

We are typically used to our actions being invariant under such co-ordinate transformations, however, our construction in the unitary gauge means this is no longer the case. This means all terms that are not contracted scalars like the Ricci scalar R, will be modified under such transformation. For example, the EFT functions $f(t), \Lambda(t), \ldots$ can be taylor expanded:

$$f(t) \to f(t + \pi(x)) = f(t) + \dot{f}(t)\pi + \frac{1}{2}\ddot{f}(t)\pi^2 + \mathcal{O}(\pi^3)$$

This also affects perturbations as $\delta A = A - \bar{A}(t) \to \delta A - \dot{\bar{A}}\pi + \ldots$ as well as operators with free indices such as g^{00}. Some examples of these transformations are [32] (see [37, 39]):

$$\begin{aligned} \delta g^{00} &\to \delta g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi \\ \delta K_{00} &\to \delta K_{00} + \mathcal{O}(\pi^2) \\ \delta K_{ij} &\to \delta K_{ij} - h_{ij}\dot{H}\pi - \partial_i\partial_j\pi + \mathcal{O}(\pi^2) \\ \delta K &\to \delta K - 3\dot{H}\pi - \frac{1}{a^2}\partial^2\pi + \mathcal{O}(\pi^2) \end{aligned} \tag{4.5}$$

Another way of phrasing the symmetry breaking formalism is to notice that a theory can be identified to break a symmetry if from the unitary gauge the Stückelberg trick produces interacting π particles

[32] These transformations can be obtained by simply considering the tensor transformation in the case of g^{00}, where as for $K_{\mu\nu}$, the change of the normal vector 4.1 also has to be considered. See the appendix of [38] for further details.

[39]. Hence, the only way preserve time translation invariance is to enforce that $f, \Lambda = \text{const}$ and all other EFT functions vanish. This is nothing more than the sub case of ΛCDM with a cosmological constant Λ.

The Stückelberg trick is the easiest way to see how this added scalar degree of freedom can manifest itself and the action 4.4 can be re-written in terms of π using the above transformations 4.5, see [40]. This form of the action is useful as it tells us how this extra scalar freedom couples to gravity. Analysis of gravitational perturbations, such as the potentials ϕ and ψ in the Newtonian gauge, allows one to explicitly see the kinetic mixing of π to gravity. Studying the equations of motion of π can also allow one to analyse the stability of the theory as we discuss in section 5.2.

4.4 Modified Friedman Equations

Since the action 4.4 offers a clear distinction between the background and the perturbations, the EoM can be studied in order to connect to other quantities of interest to background, such as the expansion $H(t)$. It will also be useful to characterize the background effects of dark energy as a fluid with a density $\rho_{\rm DE}$ and pressure $P_{\rm DE}$, so that the equation of state $\omega_{\rm DE} := \rho_{\rm DE}/P_{\rm DE}$, can be considered. This is important given how much of the observational probes centre around this quantity, as was discussed in section 2.4.1.

Analysis of the background begins by finding the equations of motion of just the first line of our action 4.4. This is quite similar to the usual derivation of Einstein's equations 1.2, with the only major difference coming from the $f(t)$ attached to R [33]. Defining the energy momentum tensor as $T_{\mu\nu} := \frac{\delta S_M}{\delta g^{\mu\nu}}$, results in the following background EoM:

$$T_{\mu\nu} = M_{\rm pl}^2 \left[G_{\mu\nu} f - \nabla_\mu \nabla_\nu f + g_{\mu\nu} \Box f\right] + (\Lambda + c g^{00}) g_{\mu\nu} - 2c\delta^0_\mu \delta^0_\nu \tag{4.6}$$

Only the background part of $T_{\mu\nu}$ is important so we can make the perfect fluid approximation and write $T_{\mu\nu} = (\rho_m + P_m)u_\mu u_\nu + g_{\mu\nu} P_m$. On the background one can use the following values due to FLRW metric with spatial curvature k (see [15]):

$$R = 6\left[\dot{H} + 2H^2 + \frac{k}{a^2}\right], \quad R_{00} = -3\frac{\ddot{a}}{a}, \quad \nabla_0 \nabla_0 f = \ddot{f}, \quad \Box f = -\ddot{f} - 3H\dot{f}$$

These can be plugged into the 00th and trace equations of 4.6 and manipulated to find the following expressions for $\Lambda(t)$ and $c(t)$ in terms of $f(t)$ and the expansion history $H(t)$:

$$\Lambda(t) = \frac{1}{2}(P_m - \rho_m) + M_{\rm pl}^2 f\left[\dot{H} + 3H^2 + 2\frac{k}{a^2} + \frac{1}{2}\frac{\ddot{f}}{f} + \frac{5}{2}H\frac{\dot{f}}{f}\right] \tag{4.7}$$

$$c(t) = \frac{1}{2}(\rho_m + P_m) + M_{\rm pl}^2 f\left[\frac{k}{a^2} - \dot{H} - \frac{1}{2}\frac{\ddot{f}}{f} + \frac{1}{2}H\frac{\dot{f}}{f}\right] \tag{4.8}$$

Conservation of the matter stress energy tensor also implies:

$$\dot{\rho}_m + 3H(\rho_m + P_m) = 0$$

[33] Usually when varying the action one can ignore $\delta R_{\mu\nu}$ as it proportional to a total derivative. However, with $f \neq \text{const}$, this results in some extra derivative terms that act on f after integration by parts.

If we want to return to a more familiar picture where dark energy is also a fluid we can define its density and pressure through the following Modified equations[34]:

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_{\text{pl}}^2 f}(\rho_m + \rho_{DE}) \tag{4.9}$$

$$\dot{H} = \frac{k}{a^2} - \frac{1}{2M_{\text{pl}}^2 f}\left(\rho_m + \rho_{DE} + P_m + P_{DE}\right) \tag{4.10}$$

Note that f should be included since it comes coupled to the Ricci scalar along side M_{pl}^2[35]. The consequences of this are that the continuity equation, which can be obtained by taking derivatives and combining 4.9 and 4.10, is not conserved like the matter equation was:

$$\dot{\rho}_{DE} + 3H\left(\rho_{DE} + P_{DE}\right) = 3M_{\text{pl}}^2\left(H^2 + \frac{k}{a^2}\right)\dot{f}$$

We can see that this is non-conservation is entirely a consequence of $f \neq \text{const}$. Comparing 4.9, 4.10 to 4.7 and 4.8 one finds the relations:

$$c(t) = \frac{1}{2}(\rho_{DE} + P_{DE}) - \frac{1}{2}M_{\text{pl}}^2\left(\ddot{f} - H\dot{f}\right)$$
$$\Lambda(t) = \frac{1}{2}(\rho_{DE} - P_{DE}) = \frac{1}{2}M_{\text{pl}}^2\left(\ddot{f} + 5H\dot{f}\right)$$

These relations tell us that out of the variables $\{c, \Lambda, f, H, \rho_{DE}, P_{DE}\}$ only two are independent. To fully determine the system only two choice have to be made. An example would be to fix the expansion history $H(t)$ and choose an equation of state for dark energy ω_{DE} (relating ρ_{DE} and P_{DE}).

5 Phenomenology with the EFTDE

The Effective Field Theory of Dark Energy (EFTDE), presented in the form of the action 4.4, allows a unifying description of dark energy and modified gravity models. The ultimate goal is to use the EFTDE to determine what models are/are-not viable by comparing the parameters to cosmological observations. The background functions (first line of 4.4) are not enough to distinguish between different dark energy models, hence, linear perturbations need to be studied. In this section we will discuss how different theories can be mapped into the EFTDE before exploring the phenomenological implications that come from the deviation from ΛCDM.

5.1 Mapping procedure

Figuring out how different theories can be expressed in terms of the parameters in 4.4 is essential to the goals of the EFTDE. For simple, minimally coupled theories ($f = \text{const}$) such as quintessence or k-essence this can be done in a straight forward manner. In the unitary gauge, functions of ϕ go to functions of time $\bar{\phi}(t)$ and hence $X \to -\frac{1}{2}\dot{\bar{\phi}}^2 g^{00} = \frac{1}{2}\dot{\bar{\phi}}^2(1 - \delta g^{00})$. From this it is clear that the quintessence action 3.1 has:

$$c(t) = -\frac{1}{2}\dot{\bar{\phi}}^2(t), \qquad \Lambda(t) = V(\bar{\phi}(t))$$

[34] The second of these might not look too familiar, but it is the same as the acceleration Friedman equation just written in terms of $\dot{H}$ instead of $\ddot{a}/a$.

[35] Alternatively, one could define ρ_{DE} and P_{DE} with out this f, in which case the continuity equation would be preserved.

Where as for k-essence 3.2 the higher order operators are also affected. If one expands:

$$P(\phi, X) = \sum_{n=1}^{\infty} \frac{p_n(\phi)}{n!} X^n \rightarrow \sum_{n=1}^{\infty} \frac{p_n(\phi)}{n!} \left(\frac{1}{2}\dot{\bar{\phi}}^2(t)\right)^n \left[1 - n\delta g^{00} + \frac{1}{2}n(n-1)(\delta g^{00})^2 + \dots\right]$$

Then it can be read off that (recall $c(t)$ is the coefficient of g^{00}, not δg^{00}):

$$c(t) - \Lambda(t) = P\left(\bar{\phi}(t), \frac{1}{2}\dot{\bar{\phi}}^2(t)\right), \quad c(t) = \frac{1}{2}\dot{\bar{\phi}}^2 \frac{\partial P}{\partial X}\left(\bar{\phi}, \frac{1}{2}\dot{\bar{\phi}}^2(t)\right), \quad M_2^4(t) = \frac{1}{2}\dot{\bar{\phi}}^2 \frac{\partial^2 P}{\partial X^2}\left(\bar{\phi}, \frac{1}{2}\dot{\bar{\phi}}^2(t)\right)$$

Beyond, minimal coupling it is clear that scalar tensor theories like 3.3 will also generate $f(t) = \frac{1}{2}\bar{\phi}(t)$ and coefficients involving the extrinsic curvature $K_{\mu\nu}$ 4.2 can be generated from higher derivative terms such as those appearing in Horndeski's theory 3.4. This can be seen in the fact that in the unitary gauge the relation between the normal vector and the ϕ 4.1 allows us to express:

$$K = \nabla_\mu n^\mu \xrightarrow[\text{gauge}]{\text{unitary}} -\nabla_\mu \left(\frac{-\nabla^\mu \phi}{\sqrt{-\nabla_\nu \phi \nabla^\nu \phi}}\right) = -\frac{3}{2}\frac{\Box \phi}{\sqrt{-\nabla_\nu \phi \nabla^\nu \phi}} + \text{tot derivative}$$

For more complicated Lagrangians, it can be easier to write the theory in the ADM formalism as there all the relevant operators have been mapped to the EFTDE, see [40] for a review.

5.2 Stability Conditions

Once a theory has been mapped to the EFTDE a whole host of options for analysing the theory are opened up. In particular, through the Stüeckelberg trick introduced in section 4.3, the dark energy scalar degree of freedom reappears as π in a manageable fashion, through the expressions in 4.5. Focusing on the kinetic π terms produced, one can then determine the stability of the given theory and hence, rule out unphysical areas of the parameter space.

There are three main types of instabilities that could plague any given theory of a scalar field that can appear after the Stüeckelberg trick:

- **Ghost Instability**: This is when the scalar field is produced with a kinetic term that has the "wrong sign":

  $$S_{EFT} \supset \int d^4x \sqrt{-g}\left[\tilde{f}(t)\partial_\mu \pi \partial_\nu \pi\right], \quad \tilde{f}(t) > 0$$

  Compare this to 3.1 for example of the usual sign. This wrong sign is a big problem for the stability of the theory, as it means these particles have negative energy. This can be inferred from calculation of the hamiltonian in terms of creation and annihilation operators, with the result being minus the usual expression A.1, meaning the creation operators create negative energy states. As a result of this, one always demands that the coefficients of the kinetic term of π remain negative.

- **Gradient/Laplace Instability**: This is essentially when the scalar has a negative speed of sound c_s^2:

  $$S_{EFT} \supset \int d^4x \sqrt{-g}\left[\tilde{f}_1(t)\dot{\pi}^2 - \tilde{f}_2(t)\nabla^2\pi\right], \quad -\frac{\tilde{f}_2(t)}{\tilde{f}_1(t)} = c_s^2 < 0$$

  Taking $c_s \simeq$ const leads to mode solutions of the form $\pi \sim e^{\pm i c_s \omega t}$, hence if $c_s^2 < 0$ there exits unbounded growing modes. Furthermore, the time scale that these modes appear for is (for high energy solutions) $\sim \frac{1}{k}$, so high energy modes contribute the most, leading to a theory that does not make any sense as an EFT.

- **Tachyonic Instability**: Here, the scalar appears to have a negative mass term, which usually indicates we are not expanding around the vacuum of the theory (like the Higgs vacuum).

$$S_{EFT} \supset \int d^4x\sqrt{-g}\left[-\frac{1}{2}\tilde{m}(t)^2\pi^2\right], \quad \tilde{m}(t)^2 < 0$$

  Again this leads to a growing mode instability, this time for low energy modes $k \to 0 \implies \pi \sim e^{\pm i\omega t} \to e^{\pm|\tilde{m}|t}$. This time however, the time scale is k independent $\sim \tilde{m}^{-1}$ and cannot get arbitrarily high. So for these low energy modes, the instability is unimportant if the time scales are long, i.e. $|\tilde{m}| \lesssim H$, we do not have to enforce $\tilde{m}(t)^2 > 0$.

5.3 Phenomenological Functions

The entire discussion of section 2 pertained to the analysis of evolution at the background level, i.e. the first order approximation. Separate to this, is the question of structure formation. This is a regime well governed by linear perturbations, where we can begin (at least in principle) to distinguish different models in the EFTDE. Central to this idea is the phenomenological functions μ, Σ and η, which we will now introduce. When analysing perturbations it is often very convenient to use the break the diffeomorphism invariance in a particular way so as to simplify the form of the metric. For us this means the choice of the Newtonian Gauge:

$$ds^2 = -\left(1+2\Phi\right)dt^2 + a(t)^2(1-2\Psi)dx^i\delta_{ij}dx^j$$

In this gauge, analysis of the scalar perturbations of matter through the energy density $\delta\rho$ and velocity δu leads to the following “Poisson” equation, (see Chapter 6 of [15]):

$$\delta\rho = 3H(\bar{\rho}+\bar{p})\delta u + \frac{1}{4\pi G a^2}\nabla^2\Psi \tag{5.1}$$

Setting $\bar{p} = 0$ for matter and using irrotational part of the peculiar velocity v [36] allows the repackaging of $\delta_m := \frac{\delta\rho_m}{\bar{\rho}_m}, \delta u$ into the comoving fractional density $\Delta_m := \delta_m + 3Hav/k$ resulting in the following Fourier space expression:

$$-\frac{k^2}{a^2}\Psi = 4\pi G\bar{\rho}_m\Delta_m \tag{5.2}$$

This equation tells us exactly how structure formation is affected by gravity in ΛCDM and general relativity. If we want to understand how different models of dark energy or modified gravity can differ from this we can parametrize deviations through the *Effective Gravitational Coupling* $\mu(\mathbf{k}, t)$, defined through:

$$-\frac{k^2}{a^2}\Phi = 4\pi G\bar{\rho}_m\mu(\mathbf{k},t)\Delta_m \tag{5.3}$$

This definition is strange, given that it is Ψ that appears in 5.2, not Φ. However, this definition is preferred, as turns out to be easier to see the observational effects of Φ as it plays the same role as the classical Newtonian potential. It is also true that the same analysis of scalar perturbations that leads to 5.1, leads to (again see chapter 6 of [15]):

$$a^2\partial_i\partial_j\pi^S = \partial_i\partial_j(\Psi - \Phi)$$

[36] $\mathbf{v}$ is the Newtonian limit counterpart of δu and is related via $\nabla^2\delta u/a = \nabla\cdot\mathbf{v}$. The irrotational part v is defined as the component parallel to $\mathbf{k}$, such that $\nabla\cdot\mathbf{v} \to kv$.

Where π^S is the scalar anisotropic inertia and is generally vanishing, at late times, for the species of matter we know to exist, which leads to $\Phi \simeq \Psi$. Deviations from this can be parametrized by the *Gravitational Slip Parameter*:

$$\eta := \frac{\Psi}{\Phi} \tag{5.4}$$

However, since measurements of lensing measure the combination $\Phi+\Psi$, sometimes known as the lensing or Weyl potential, it is more convenient to define the *Light Deflection Parameter* $\Sigma(\mathbf{k}, t)$, through:

$$-\frac{k^2}{a^2}(\Phi + \Psi) = 8\pi G\bar{\rho}_m \Sigma(\mathbf{k}, t)\Delta_m \tag{5.5}$$

These parameters are not independent and can be related through $\Sigma = 1/2(1+\eta)\mu$. The case of ΛCDM with no modified gravity corresponds to $\Sigma = \mu = \eta = 1$ as then both 5.5 and 5.3 reduced to 5.2.

Analytic forms these functions are not easily computable from first principles. However, within the *Quasi Static* (QS) approximation[37], allows the computation of Σ, μ and η for several theories. This approximation is heavily aided by the use of the EFT formalism outlined above and has allowed the calculation of Σ, μ and η for general theories such as Horndeski 3.4, see [41]. Outside of this approximation, the EFTDE can help compute Σ, μ and η numerically by calculating the full evolution of perturbations.

5.3.1 The α Basis

There is another set of time dependent functions, similar to the EFT functions 4.2.1, but more focused on describing the phenomenological effects[38] of linear perturbations on large scale structure [43]. To describe general scalar tensor theories such as Horndeski's 3.4 and beyond, 7 functions, collectively known as the α-basis are needed. They are related to the EFT functions in 4.4, by the following expressions:

$$\begin{gathered}\alpha_M = \frac{1}{H}\frac{dM^2}{d\ln t}, \quad \alpha_T(t) = \frac{\bar{M}_3^2}{M^2}, \quad \alpha_B(t) = -\frac{M_{\text{pl}}^2 \dot{f} + \bar{m}_1^3}{HM^2}, \quad \alpha_K(t) = \frac{2c + 4M_2^4}{H^2M^2} \\ \alpha_{K_2}(t) = \frac{8m_2^2}{H^2M^2}, \quad \alpha_H(t) = \frac{2\mu_1^2 + \bar{M}_3^2}{M^2}, \quad \alpha_B^{GLPV}(t) = \frac{\bar{M}_3^2 + \bar{M}_2^2}{M^2}\end{gathered} \tag{5.6}$$

Where we have defined the *Effective Planck Mass* $M^2 = M_{\text{pl}}^2 f - \bar{M}_3^2$. These functions represent:

- α_M: The *Running of the Planck Mass.*
- α_B: The *Braiding function* (mixing between gravity and the dark energy scalar).
- α_T: The *Tensor Speed Excess* (how the speed of gravitational waves differs from c).
- α_K: The *Kineticity* (affect the speed of propagation of the scalar).
- α_{K_2}: The *Lorentz violating Kineticity.*
- α_H: The *Horndeski Parameter* (measures deviations from Horndeski's theory 3.4).
- α_B^{GLPV}: The *GLPV Braiding function* (extends braiding to theories beyond GLPV).

[37] Where one neglects time derivative perturbations, assuming they evolve on Hubble time scales (eg $\dot{\Phi} \simeq H\Phi << \partial_i\Phi$).
[38] For example, observations of the gravitational wave GW170817 and its visible counterpart, constrain $\alpha_T < 10^{-15}$ [42].

5.4 Observations of Cosmological Perturbations

These functions that we have collected throughout this essay, the EFT functions $f, c, \Lambda, M_2, \ldots$ in 4.4, the alpha basis $\alpha_M, \alpha_B, \ldots$ 5.6 and the phenomenological functions Σ, η and μ in 5.3, put us in an ideal position to test ΛCDM and GR with observations of cosmological perturbations. The best constraints we have so far come again from DESI [44]. Using, full shape modelling of the power spectrum and clustering measurements from the DESI observations, in combination with CMB, galaxy weak lensing and supernovae data, DESI have begun to give us a picture of what these functions look like.

Since, these are functions of time, a parametrization is needed to fit for constant parameters. For this it is assumed that the functions are proportional to the energy density of dark energy. So for example for μ from 5.3, Σ from 5.5[39] and the two of the α functions α_B, α_M from 5.6:

$$\mu(a) = 1 + \mu_0 \frac{\Omega_{DE}(a)}{\Omega_\Lambda}, \quad \Sigma(a) = 1 + \Sigma_0 \frac{\Omega_{DE}(a)}{\Omega_\Lambda}$$
$$\alpha_M = c_M \Omega_{DE}(a), \quad \alpha_B = c_B \Omega_{DE}(a)$$

Given these parametrizations, DESI can find fits by numerically simulating the evolution of perturbations and checking the results against observations to see how well the statistics match. The results of these fittings are shown in the contour plots in Figure 10. These results show that the functions μ and Σ are fully consistent with their expected GR values of 1 as $\mu_0 = 0.05 \pm 0.22$ and $\Sigma_0 = 0.008 \pm 0.045$. Where as for α_M and α_B there is a slight preference for $\alpha_B \neq 0$, while remaining consistent with $\alpha_M = 0$ as $c_B = 0.92 \pm 0.33$ and $c_M = 1.05 \pm 0.96$. This analysis was done assuming a ΛCDM background, but similar analysis can be done using a $\omega_0\omega_a$CDM background. This results in very similar results from the modified gravity parameter, however, it is still interesting that the preference for time evolving equation of state. We have not mentioned the EFT functions here directly, though the data shows similar results to that of Σ, μ and α_M with the data being consistent with their values in GR [44].

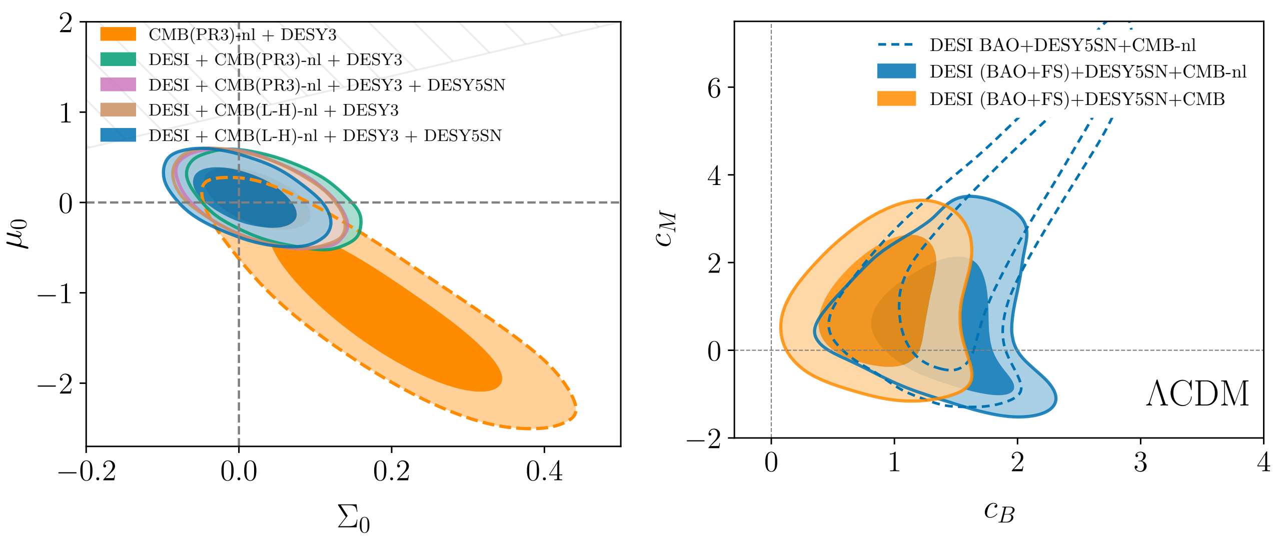

Figure 10: Constraints on cosmological perturbation functions. (Left) Fits for the phenomenological parameters Σ_0 and μ_0, showing strong consistency with GR. (Right) Fits for the EFT parameters c_B and c_M, indicating a slight preference for non-zero α_B.

[39]Note in the case of μ and Σ, the **k** dependence is also dropped.

Conclusions

The long standing tensions between vacuum energy and gravity continue to challenge the foundations of quantum field theory. While the cosmological constant Λ remains the simplest solution, recent observations from probes like DESI have placed its permanence under renewed scrutiny. We have reached a critical juncture where the Effective Field Theory of Dark Energy serves as the definitive diagnostic tool. If the time-evolving parameter ω_a and the phenomenological functions governing gravitational perturbations remain zero, dynamical dark energy may be effectively ruled out. Conversely, the detection of non-zero parameters hints of which may already be emerging, would provide the necessary roadmap to isolate the specific physical mechanisms driving the universe's accelerated expansion, redefining our understanding of gravity on a cosmic scale.

A Vacuum Energy

To see where the energy of the vacuum comes from consider a scalar field (though the same argument works for higher spin). For a free scalar field [40] one can decompose the field into a sum of momentum solutions to the Klein Gordon equation with the coefficients being creation and annihilation operators. From the Lagrangian one can then compute the Hamiltonian in terms of these operators [45]:

$$H = \int \frac{d^3p}{(2\pi)^3}\omega_{\mathbf{p}}\left(a^\dagger_{\mathbf{p}}a_{\mathbf{p}} + \frac{1}{2}\left[a_{\mathbf{p}}, a^\dagger_{\mathbf{p}}\right]\right) + \int d^3x V_0 \tag{A.1}$$

Where we have added an arbitrary constant potential [41]. The first term is what we would expect and vanishes in the vacuum expectation value. However, the second term by the commutation relations evaluates to $\left[a_{\mathbf{p}}, a^\dagger_{\mathbf{p}}\right] = (2\pi)^3\delta^{(3)}(0)$, where $\delta^{(3)}(\mathbf{x})$ is the Dirac delta function. This is a very divergent quantity, but since it is in momentum space it can be recognised as the volume of space as $(2\pi)^3\delta^{(3)}(0) = (2\pi)^3\int d^3xe^0 = (2\pi)^3V$. This means the energy density of the vacuum is:

$$\begin{aligned}\langle\rho\rangle = \frac{1}{V}\langle 0|H|0\rangle = \frac{1}{2}\int d^3p\omega_{\mathbf{p}} + V_0 = 2\pi\int_0^{\Lambda_{\mathrm{UV}}} dpp^2\sqrt{p^2+m^2} + V_0 \\ = V_0 + \frac{\pi}{2}\left[\Lambda^4_{\mathrm{UV}} + m^2\Lambda^2_{\mathrm{UV}} + \frac{1}{8}\left[1 - 4\ln(2) + 2\ln\left(\frac{m^2}{\Lambda^2_{\mathrm{UV}}}\right)\right]m^4\right] + \mathcal{O}(\frac{m^6}{\Lambda^2_{\mathrm{UV}}})\end{aligned} \tag{A.2}$$

A.1 Vacuum Energy in Curved Spacetime

In curved spacetime the metric $g_{\mu\nu}$ is no longer the Minkowski metric and hence the measure of the matter actions get modified $d^4x \to d^4x\sqrt{-g}$ in order for the lagrangian to be a scalar. We can then consider what happens we integrate out some scalar field φ from the path integral $Z[g,\varphi]$. In flat spacetime the field φ is not coupled to the metric and hence there is no issue in intgrating out the fields they simply give an over all contribution to the normalization of the $Z[g]$ (which does not affect the physics). However, in curved spacetime the coupling to gravity through $\sqrt{-g}$ means there is an effective contribution to the cosmological constant that can be calculated to one loop:

$$Z[g] = \int \mathcal{D}\varphi Z[g,\varphi] = \int \mathcal{D}\varphi e^{i\int d^4x\sqrt{-g}\mathcal{L}_\varphi}$$

[40] A similar vacuum energy should also appear in interacting theories as the fock space at a fixed time is identical to some free theory.

[41] Usually in flat spacetime this is not included as it has no impact on any physical observables, but it can be considered to contribute to the cosmological constant.

For simplicity we can take φ to be a free scalar field so that $\mathcal{L}_\varphi = -\frac{1}{2}\varphi(\Box + m^2)\varphi$, in which case the integral is gaussian and we can extend the integral identity (see pg 255 of [29]):

$$\prod_i \int_{-\infty}^{\infty} dp_i \exp[-\frac{1}{2}\mathbf{p}^T A\mathbf{p}] = \sqrt{\frac{(2\pi)^n}{\det A}}$$

To the operator case to get:

$$Z[g] = \mathcal{N}\left[\det\left(\int d^4x\sqrt{-g}(\Box + m^2)\right)\right]^{-\frac{1}{2}}$$

Where $\mathcal{N}$ is some normalisation. The effective action for this path integral can be defined through $e^{i\Gamma} := Z[g]$ so that:

$$\Gamma = \frac{i}{2}\ln\left[\det\left(\int d^4x\sqrt{-g}(\Box + m^2)\right)\right] = \frac{i}{2}\mathrm{Tr}\left[\ln\left(\int d^4x\sqrt{-g}(\Box + m^2)\right)\right]$$

Where we have used the identity $\ln\det A = \mathrm{Tr}\ln A$ and have ignored a genuine constant. The trace here is an operator trace over some basis of states $|x\rangle$. To write this in a more appealing form note that we can take a derivative wrt m^2 of this operator:

$$\begin{aligned}\frac{\partial}{\partial m^2}\ln\left(\int d^4x\sqrt{-g}(\Box + m^2)\right) &= \int d^4x\sqrt{-g}\frac{1}{\Box + m^2} = \int d^4x\sqrt{-g}\int \frac{d^4k}{(2\pi)^4}\frac{1}{-k^2 + m^2}e^{ik\cdot x}\\ \implies \ln\left(\int d^4x\sqrt{-g}(\Box + m^2)\right) &= \int d^4x\sqrt{-g}\int\frac{d^4k}{(2\pi)^4}\ln(-k^2 + m^2)e^{ik\cdot x}\end{aligned}$$

Where we have performed integration wrt m^2 to get the second line. We can then perform this integral using a wick rotation and a cutoff Λ_{UV}:

$$\Gamma \simeq \frac{1}{16\pi^2}\int d^4x\sqrt{-g}\left[\Lambda_{\mathrm{UV}}^4(4\ln(\Lambda_{\mathrm{UV}}) - 1) + 4m^2\Lambda_{\mathrm{UV}}^2 + \frac{1}{8}\left[-1 + 2\ln\left(\frac{m^2}{\Lambda_{\mathrm{UV}}^2}\right)\right]m^4 + \dots\right]$$

Which are the same sort of contribution that come from the flat space vacuum energy A.2. Hence the integrating out of any fields will produce a large contribution from radiative corrections to the cosmological constant.

B Screening Potential

Consider the following Lagrangian:

$$\mathcal{L} = -\frac{1}{2}Z^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) + g(\phi)T^\mu_{\ \mu}$$

Here $Z^{\mu\nu}$ could be a function of ϕ and its derivatives and $T^\mu_{\ \mu}$ is the stress energy tensor of regular matter. The idea then is to make a mean field approximation, where we split the stress energy tensor and the field into background and perturbation: $\phi = \bar{\phi} + \varphi$ and $T^{\mu\nu} = \bar{T}^{\mu\nu} + t^{\mu\nu}$. The equations of motion can then be solved separately for $\bar{\phi}$ and φ, with the equation of motion for φ coming out to:

$$Z(\bar{\phi})\left(\ddot{\varphi} - c_s^2(\bar{\phi})\nabla^2\varphi\right) + m^2(\bar{\phi})\varphi = g'(\bar{\phi})t^\mu_{\ \mu} \tag{B.1}$$

Here we have assumed that $Z(\bar{\phi})$ varies negligibly spatially on the scale of fifth force interactions. $m^2(\bar{\phi}) := \frac{\partial^2 V}{\partial\phi^2}\big|_{\bar{\phi}}$ is the mass. To see what sort of force this gives rise to we can simply calculate the amplitude of the diagram in Figure 9. The propagator for the free scalar field (from LHS of B.1) is simply:

$$G(k) = \frac{i}{Z[(k^0)^2 - c_s^2\mathbf{k}^2] - m_\phi^2},$$

and the interactions will come through the term $\varphi g'(\bar{\phi})t^\mu{}_\mu \subset g(\phi)T^\mu{}_\mu$, meaning the vertices are each proportional to $g'(\bar{\phi})$[42]. Lastly, to arrive at a potential we look at the non-relativistic limit where the incoming momentum are $p = (m, \mathbf{p}),\ q = (m, \mathbf{q})$ so that the amplitude is simply $\mathcal{M}a \simeq g'(\bar{\phi})^2 G(p-q)$, with $(p-q)^2 = |\mathbf{p}-\mathbf{q}|^2$. Following [45] we can calculate the potential as using the Born approximation the non relativistic potential is simply the Fourier transform of the amplitude in momentum space. So inverting this Fourier transform we find the following potential:

$$V(r) \propto \frac{g'(\bar{\phi})^2}{Z(\bar{\phi})c_s^2(\bar{\phi})}\frac{1}{r}e^{-\frac{m(\bar{\phi})}{\sqrt{Z(\bar{\phi})}c_s(\bar{\phi})}r} \tag{B.2}$$

C Jordan and Einstein Frames

When constructing theories of modified gravity that introduce non minimal couplings to the extra degrees of freedom (eg scalar tensor theories 3.3 modify $\frac{1}{2}M_{\rm pl}^2 R$ to $\frac{1}{2}M_{\rm pl}^2\phi R$.), one is naturally faced with a choice for the metric $g_{\mu\nu}$. The choice arises due to the fact that a *Weyl Transformation*, which is essentially a re-definition of the metric $\tilde{g}_{\mu\nu} = f(\phi)g_{\mu\nu}$, can always be made such that the non minimal part of the coupling is removed (eg $\phi R \to R$), though this will alter the other parts of the action containing $g_{\mu\nu}$. This leads naturally to two choices of metric/frame:

- The *Jordan Frame* keeps the non minimal coupling, so that the metric that couples to other matter remains the same as that in the Ricci scalar.
- The *Einstein Frame* uses the Weyl transformation to remove the non minimal coupling and hence the metric that couples to matter is no longer the same. As a consequence particles no longer travel along geodesics and the Weak Equivalence Principle (WEP) is broken.

When building the EFTDE in section 4, we choose the Jordan Frame as the matter action $S_m[g_{\mu\nu}, \psi_i]$ is present and observations of WEP are very constraining. This is unlike the EFT of inflation [33], which used the Einstein frame as regular matter can be neglected[43].

D EFTDE Action Terms

Here we give a brief explanation of the apparent absence of certain terms from the action 4.4.

- $K = K_{\mu\nu}g^{\mu\nu} = \nabla_\mu n^\mu$: This term is absent as it contains a derivative that can simply be integrated by parts onto its corresponding EFT function $g(t)$. Since the EFT functions are only functions of time $\partial_\mu g \propto n_\mu$, and hence $g(t)K \sim \tilde{g}(t)n^\mu n_\mu$ is not a new term. This also means δK cannot appear in the action.
- K^{00}: This term cannot appear as $K^{\mu\nu}n_\mu n_\nu = 0$.

[42]We are ignoring any intricacies involving the external spinor legs of this diagram.
[43]Hence, no function like $f(t)$ in 4.4 is needed in front of R in the action.

- $\nabla_\mu \ldots \nabla_\nu A$: Higher co-variant derivatives of any operator can be integrated by parts on the EFT functions to generate terms comprised of n_μ and $K_{\mu\nu}$.
- R^{00}: Can be written in terms of $K_{\mu\nu}$ and co-variant derivatives of $K_{\mu\nu}$, see the Appendix of [33] for further details.
- $\delta K_{\mu\nu} := K_{\mu\nu} - \bar{K}_{\mu\nu}$: This term could only come from $K_{\mu\nu}g^{\mu\nu} = K$, which we ruled out above, $\bar{R}^{\mu\nu}\delta K_{\mu\nu} \subset K_{\mu\nu}R^{\mu\nu}$ or $\bar{K}^{\mu\nu}\delta K_{\mu\nu} \subset K_{\mu\nu}K^{\mu\nu}$. However, the background quantities $\bar{R}^{\mu\nu}$ and $\bar{K}^{\mu\nu}$ are simply functions of t, n_μ and $g_{\mu\nu}$, so integration by parts will always lead to the terms found in the first line of 4.4.

For further details on how higher order curvature terms do not contribute see Appendix A of [37].

References

[1] Steven Weinberg. *Steven Weinberg: A Life in Physics*. Cambridge University Press, Nov. 2024. ISBN: 978-1-009-51348-7, 978-1-009-51347-0. DOI: 10.1017/9781009513487.

[2] Albert Einstein. “Cosmological Considerations in the General Theory of Relativity”. In: *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* 1917 (1917).

[3] Steven Weinberg. “The Cosmological Constant Problem”. In: *Rev. Mod. Phys.* 61 (1989). Ed. by Jong-Ping Hsu and D. Fine. DOI: 10.1103/RevModPhys.61.1.

[4] Eugenio Bianchi and Carlo Rovelli. “Why all these prejudices against a constant?” In: (Feb. 2010). arXiv: 1002.3966 [astro-ph.CO].

[5] N. Aghanim et al. “Planck 2018 results. VI. Cosmological parameters”. In: *Astron. Astrophys.* 641 (2020). [Erratum: Astron.Astrophys. 652, C4 (2021)]. DOI: 10.1051/0004-6361/201833910. arXiv: 1807.06209 [astro-ph.CO].

[6] CP Burgess. “The cosmological constant problem: why it’s hard to get dark energy from micro-physics”. In: *arXiv preprint arXiv:1309.4133* (2013).

[7] Steven Weinberg. “Anthropic Bound on the Cosmological Constant”. In: *Phys. Rev. Lett.* 59 (1987). DOI: 10.1103/PhysRevLett.59.2607.

[8] Hugo Martel, Paul R. Shapiro, and Steven Weinberg. “Likely values of the cosmological constant”. In: *Astrophys. J.* 492 (1998). DOI: 10.1086/305016. arXiv: astro-ph/9701099.

[9] Adam G. Riess et al. “Observational evidence from supernovae for an accelerating universe and a cosmological constant”. In: *Astron. J.* 116 (1998). DOI: 10.1086/300499. arXiv: astro-ph/9805201.

[10] J. A. Peacock. *Cosmological Physics*. Cambridge University Press, 1998.

[11] H. E. S. Velten, R. F. vom Marttens, and W. Zimdahl. “Aspects of the cosmological “coincidence problem””. In: *Eur. Phys. J. C* 74.11 (2014). DOI: 10.1140/epjc/s10052-014-3160-4. arXiv: 1410.2509 [astro-ph.CO].

[12] Steven Weinberg. *Cosmology*. 2008. ISBN: 978-0-19-852682-7.

[13] S. Perlmutter et al. “Measurements of Ω and Λ from 42 High Redshift Supernovae”. In: *Astrophys. J.* 517 (1999). DOI: 10.1086/307221. arXiv: astro-ph/9812133.

[14] T. M. C. Abbott et al. “The Dark Energy Survey: Cosmology Results with ∼1500 New High-redshift Type Ia Supernovae Using the Full 5 yr Data Set”. In: *Astrophys. J. Lett.* 973.1 (2024). DOI: 10.3847/2041-8213/ad6f9f. arXiv: 2401.02929 [astro-ph.CO].

[15] Daniel Baumann. *Cosmology*. Cambridge University Press, July 2022. ISBN: 978-1-108-93709-2, 978-1-108-83807-8. DOI: 10.1017/9781108937092.

[16] P. Meszaros. "The behaviour of point masses in an expanding cosmological substratum". In: *Astron. Astrophys.* 37 (1974).

[17] Will J. Percival et al. "The shape of the SDSS DR5 galaxy power spectrum". In: *Astrophys. J.* 657 (2007). DOI: 10.1086/510615. arXiv: astro-ph/0608636.

[18] A. G. Adame et al. "DESI 2024 III: baryon acoustic oscillations from galaxies and quasars". In: *JCAP* 04 (2025). DOI: 10.1088/1475-7516/2025/04/012. arXiv: 2404.03000 [astro-ph.CO].

[19] Michel Chevallier and David Polarski. "Accelerating universes with scaling dark matter". In: *Int. J. Mod. Phys. D* 10 (2001). DOI: 10.1142/S0218271801000822. arXiv: gr-qc/0009008.

[20] Eric V. Linder. "Exploring the expansion history of the universe". In: *Phys. Rev. Lett.* 90 (2003). DOI: 10.1103/PhysRevLett.90.091301. arXiv: astro-ph/0208512.

[21] Eric V. Linder. "The Dynamics of Quintessence, The Quintessence of Dynamics". In: *Gen. Rel. Grav.* 40 (2008). DOI: 10.1007/s10714-007-0550-z. arXiv: 0704.2064 [astro-ph].

[22] DESI Collaboration et al. "DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations". In: *Journal of Cosmology and Astroparticle Physics* 2025.2 (Feb. 1, 2025). DOI: 10.1088/1475-7516/2025/02/021. arXiv: 2404.03002[astro-ph].

[23] Tian-Nuo Li et al. "Robust evidence for dynamical dark energy in light of DESI DR2 and joint ACT, SPT, and Planck data". In: *Physics of the Dark Universe* 52 (June 2026). DOI: 10.1016/j.dark.2026.102254. arXiv: 2511.22512[astro-ph].

[24] B. Popovic et al. *The Dark Energy Survey Supernova Program: A Reanalysis Of Cosmology Results And Evidence For Evolving Dark Energy With An Updated Type Ia Supernova Calibration*. Version Number: 3. 2025. DOI: 10.48550/ARXIV.2511.07517.

[25] William Giarè et al. "An overview of what current data can (and cannot yet) say about evolving dark energy". In: *Physics of the Dark Universe* 48 (May 2025). DOI: 10.1016/j.dark.2025.101906. arXiv: 2502.10264[astro-ph].

[26] Austin Joyce et al. "Beyond the Cosmological Standard Model". In: *Phys. Rept.* 568 (2015). DOI: 10.1016/j.physrep.2014.12.002. arXiv: 1407.0059 [astro-ph.CO].

[27] Steven Weinberg. "Photons and Gravitons in S -Matrix Theory: Derivation of Charge Conservation and Equality of Gravitational and Inertial Mass". In: *Physical Review* 135.4 (Aug. 24, 1964). DOI: 10.1103/PhysRev.135.B1049. URL: https://link.aps.org/doi/10.1103/PhysRev.135.B1049.

[28] Joseph Polchinski. *The Cosmological Constant and the String Landscape*. Apr. 21, 2006. DOI: 10.48550/arXiv.hep-th/0603249. arXiv: hep-th/0603249.

[29] Matthew D. Schwartz. *Quantum Field Theory and the Standard Model*. Cambridge University Press, Mar. 2014. ISBN: 978-1-107-03473-0, 978-1-107-03473-0. DOI: 10.1017/9781139540940.

[30] Clare Burrage and Peter Millington. "Higgs-Induced Screening Mechanisms in Scalar-Tensor Theories". In: *Annals of the New York Academy of Sciences* 1531.1 (Jan. 2024). DOI: 10.1111/nyas.15092. arXiv: 2310.12071[hep-ph].

[31] Timothy Clifton et al. "Modified gravity and cosmology". In: *Physics Reports* 513.1 (Mar. 2012). DOI: 10.1016/j.physrep.2012.01.001.

[32] Claudia de Rham and Andrew J. Tolley. "The Encyclopedia of Cosmology: Volume 1: Modified Gravity". In: *The Encyclopedia of Cosmology*. Ed. by Giovanni G. Fazi. World Scientific, Nov. 2023. DOI: 10.1142/13578-vol1.

[33] Clifford Cheung et al. "The Effective Field Theory of Inflation". In: *Journal of High Energy Physics* 2008.3 (Mar. 6, 2008). DOI: 10.1088/1126-6708/2008/03/014. arXiv: 0709.0293[hep-th].

[34] Steven Weinberg. "Effective Field Theory for Inflation". In: *Physical Review D* 77.12 (June 27, 2008). DOI: 10.1103/PhysRevD.77.123541. arXiv: 0804.4291[hep-th].

[35] Robert M. Wald. *General relativity*. Chicago: University of Chicago Press, 1984, p. 258. ISBN: 978-0-226-87032-8 978-0-226-87033-5.

[36] Steven Weinberg and Robert V Wagoner. *Gravitation and cosmology: principles and applications of the general theory of relativity*. 1973.

[37] Giulia Gubitosi, Federico Piazza, and Filippo Vernizzi. "The Effective Field Theory of Dark Energy". In: *Journal of Cosmology and Astroparticle Physics* 2013.2 (Feb. 19, 2013). DOI: 10.1088/1475-7516/2013/02/032. arXiv: 1210.0201[hep-th].

[38] Jolyon Bloomfield et al. "Dark energy or modified gravity? An effective field theory approach". In: *Journal of Cosmology and Astroparticle Physics* 2013.8 (Aug. 7, 2013). DOI: 10.1088/1475-7516/2013/08/010.

[39] Federico Piazza and Filippo Vernizzi. "Effective Field Theory of Cosmological Perturbations". In: *Classical and Quantum Gravity* 30.21 (Nov. 7, 2013). DOI: 10.1088/0264-9381/30/21/214007. arXiv: 1307.4350[hep-th].

[40] Noemi Frusciante and Louis Perenon. "Effective field theory of dark energy: A review". In: *Physics Reports* 857 (May 2020). DOI: 10.1016/j.physrep.2020.02.004.

[41] Levon Pogosian and Alessandra Silvestri. "What can Cosmology tell us about Gravity? Constraining Horndeski with Sigma and Mu". In: *Physical Review D* 94.10 (Nov. 7, 2016). DOI: 10.1103/PhysRevD.94.104014. arXiv: 1606.05339[astro-ph].

[42] Paolo Creminelli and Filippo Vernizzi. "Dark Energy after GW170817 and GRB170817A". In: *Physical Review Letters* 119.25 (Dec. 18, 2017). DOI: 10.1103/PhysRevLett.119.251302. arXiv: 1710.05877[astro-ph].

[43] Emilio Bellini and Ignacy Sawicki. "Maximal freedom at minimum cost: linear large-scale structure in general modifications of gravity". In: *Journal of Cosmology and Astroparticle Physics* 2014.7 (July 1, 2014). DOI: 10.1088/1475-7516/2014/07/050.

[44] M. Ishak et al. "Modified Gravity Constraints from the Full Shape Modeling of Clustering Measurements from DESI 2024". In: *Journal of Cosmology and Astroparticle Physics* 2025.9 (Sept. 1, 2025). DOI: 10.1088/1475-7516/2025/09/053. arXiv: 2411.12026[astro-ph].

[45] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to quantum field theory*. Reading, USA: Addison-Wesley, 1995. ISBN: 978-0-201-50397-5, 978-0-429-50355-9, 978-0-429-49417-8. DOI: 10.1201/9780429503559.