Testing theories of gravity using pulsars

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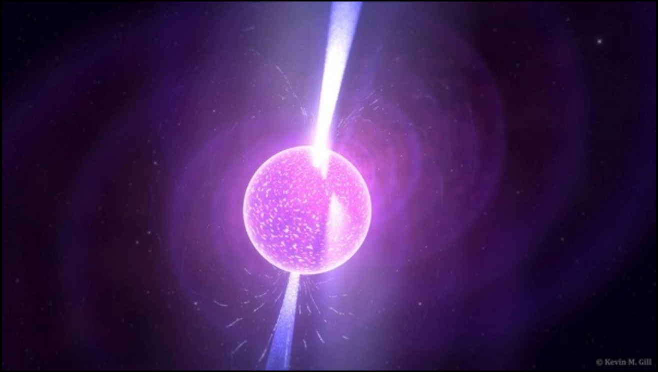
What is a pulsar?

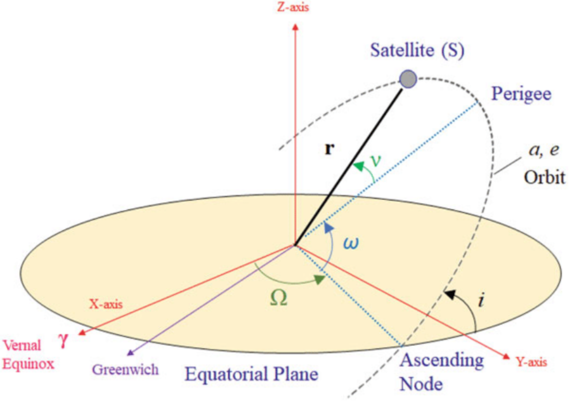
Pulsars are rapidly-rotating and highly-magnetised neutron stars that emit radiation from their poles.



Why useful?: They provide tests of strong-field gravity in ways impossible on Earth-bound laboratories or even in our Solar system.

Keplerian parameters and Binary systems

There are 6 parameters needed to fully define a binary system: a and e for the elliptical orbit, Ω and i for the orientation of the orbital plane and ω for the orientation of the ellipse in the orbital plane.



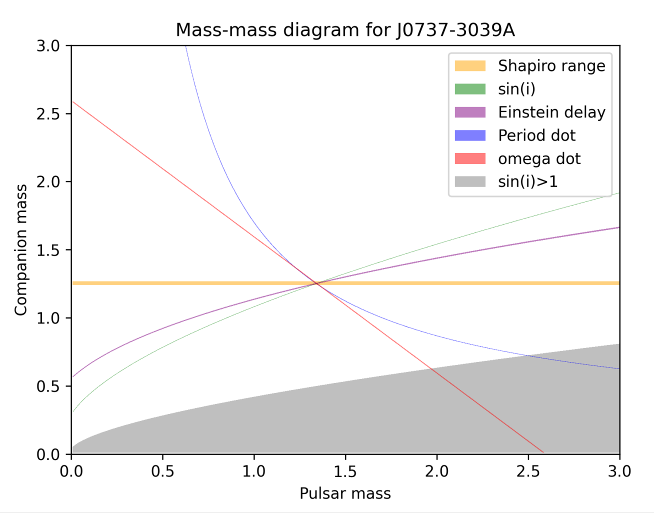
Post-Keplerian parameters

Due to Various effects of Gravity including space-time geometry around large mass objects and the emission of gravitational waves. Keplerian parameters are not stable.

This is very evident in pulsar binaries. Here the timing model of how the system changes is parameterized by post Keplerian (PK) parameters. For example in General relativity:

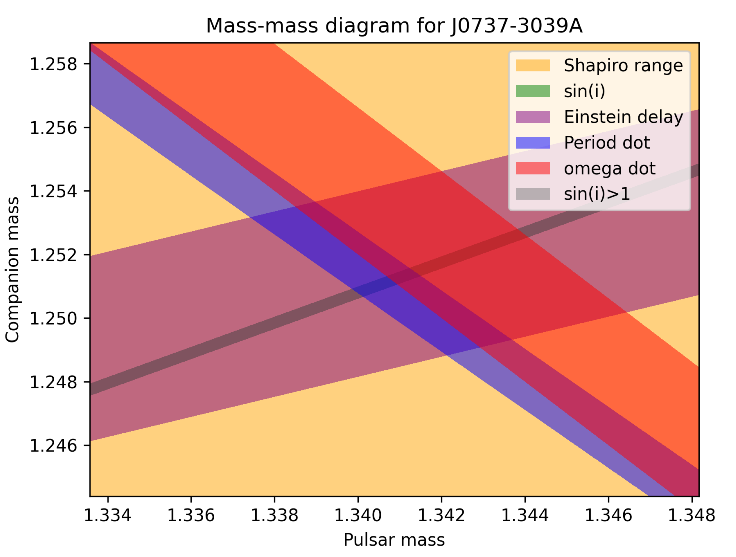
- $\dot{\omega}$ the periastron advance
- $\dot{P}_b$ the orbital decay
- γ Einstein delay
- r Shapiro range
- $s = sin(i)$ Shapiro shape

Mass-Mass diagrams

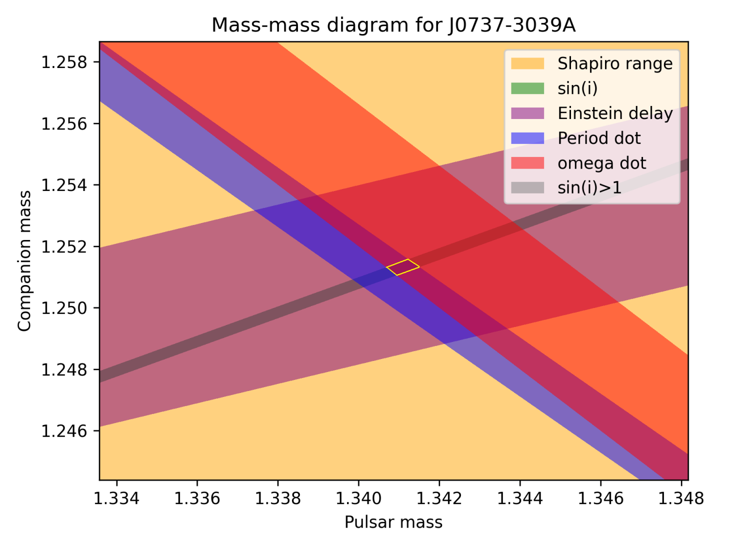


For N measured parameters provides N-2 tests of the given theory

Mass-Mass diagrams



Mass-Mass diagrams



PK parameters in GR

- $\dot{\omega} = 3T_{\odot}^{2/3}\left(\frac{2\pi}{P_b}\right)^{\frac{5}{3}} \frac{(m_p+m_c)^{\frac{2}{3}}}{(1-e^2)}$,
- $\gamma = T_{\odot}^{2/3}\left(\frac{2\pi}{P_b}\right)^{-\frac{1}{3}} e \frac{m_c(m_p+2m_c)}{(m_p+m_c)^{\frac{4}{3}}}$
- $r = T_{\odot} m_c$
- $s = sin(i) = T_{\odot}^{-\frac{1}{3}}\left(\frac{2\pi}{P_b}\right)^{-\frac{2}{3}} \frac{x(m_p+m_c)^{\frac{2}{3}}}{m_c}$
- $\dot{P}_b = -\frac{192}{5} T_{\odot}^{\frac{5}{3}}\left(\frac{2\pi}{P_b}\right)^{\frac{5}{3}} \frac{(1+(\frac{73}{24})e^2+(\frac{37}{96})e^4)}{(1-e^2)^{\frac{7}{2}}} \frac{m_p m_c}{(m_p+m_c)^{\frac{1}{3}}}$

Other Theories of Gravity

- Jordan–Fierz-Brans–Dicke theory: A scalar tensor theory which means in addition to there is a long ranged-massless scalar field ϕ which has the physical effect of changing the effective gravitational constant from place to place.
- TeVeS: Tensor-Vector-Scalar gravity: Which attempts to generalize MOND (modified Newtonian dynamics). A theory that accounts for the flat rotation curves of galaxies, instead of attributing it to the presence of dark matter.

How do the equations change?

For example for Jordan-Fierz-Brans-Dicke theory the action changes due to this scalar field

$$S = \frac{c^3}{16\pi G}\int g_*^{1/2}(R_* - 2g_*^{\mu\nu}\partial_\mu\phi\partial_\nu\phi)d^4x + S_m[\psi_m; A^2(\psi)g^*_{\mu\nu}]$$

This then leads to changes in the rotational period of the pulsar, with terms that would not be added in GR:

$$\dot{P} = \dot{P}_\phi^{Monopole} + \dot{P}_\phi^{Dipole} + \dot{P}_\phi^{Quadrapole} + \dot{P}_g^{Quadrapole} + \mathcal{O}(\frac{1}{c^7})$$

And the Gravitational constant G becomes $G_{pc} = G(1 + \alpha_p\alpha_c)$ where α_p and α_c are the coupling of matter to the scalar field for the pulsar and companion respectively.

Jordan-Fierz-Brans-Dicke theory

For example:

$$
\begin{aligned}
\dot{P}_{\varphi^*}^{\mathrm{Dipole}} = & -\frac{2\pi}{1+(\alpha_{\mathrm{p}}\alpha_{\mathrm{c}})}\nu\left(\frac{G_{\mathrm{pc}}Mn}{c^3}\right)\frac{1+\frac{e^2}{2}}{(1-e^2)^{5/2}}(\alpha_{\mathrm{p}}-\alpha_{\mathrm{c}})^2[1+\mathcal{O}\left(\frac{1}{c^2}\right)] \\
& -\frac{4\pi}{1+(\alpha_{\mathrm{p}}\alpha_{\mathrm{c}})}\nu\left(\frac{G_{\mathrm{pc}}Mn}{c^3}\right)^{5/3}\frac{1}{(1-e^2)^{7/2}} \\
& \times(\alpha_{\mathrm{p}}-\alpha_{\mathrm{c}})\left[\frac{8}{5}\left(1+\frac{31e^2}{8}+\frac{19e^4}{32}\right)(\alpha_{\mathrm{p}}X_{\mathrm{p}}+\alpha_{\mathrm{c}}X_{\mathrm{c}})(X_{\mathrm{p}}-X_{\mathrm{c}})\right. \\
& \left.+(1+3e^2+\frac{3e^4}{8})\frac{(\beta_{\mathrm{c}}\alpha_{\mathrm{p}}X_{\mathrm{p}}-\beta_{\mathrm{p}}\alpha_{\mathrm{c}}X_{\mathrm{c}})}{1+(\alpha_{\mathrm{p}}\alpha_{\mathrm{p}})}\right]+\mathcal{O}\left(\frac{1}{c^7}\right)
\end{aligned}
$$

Thanks for Listening