Dynamical Dark Energy from Causal Set Theory

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Outline

1. **Dark Energy**
2. **Causal Set Theory**
3. **Everpresent Lambda**

Dark Energy

What is Dark Energy?

- Dark Energy is the name we give to the source of the accelerated expansion of our universe.
- In our current best cosmological models (ΛCDM) Dark Energy makes up almost 70% of the energy density of the universe, yet we have no idea what it is!
- Our Current best explanation is a cosmological constant Λ in the Einstein Hilbert action:

$$S_{EH} = \frac{1}{16\pi G}\int (R - 2\Lambda)\sqrt{-g} \implies R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- This works well as in a FLRW universe, it gives rise to a constant energy density (no matter the size of the universe) and hence the scale factor grows as $a(t) \sim e^{Ht}$.

Problems with the Cosmological Constant

- Naive expectation: Cosmological constant is due to the energy density of the vacuum, which, by Lorentz invariance must have a stress energy tensor of the form:

$$\langle T_{\mu\nu} \rangle = - \langle \rho \rangle g_{\mu\nu}$$

- But from Quantum Field Theory we would expect a vacuum energy of:

$$\langle \rho \rangle = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \omega_p = \frac{1}{4\pi^2} \int_0^{k_{\max}} dk k^2 \sqrt{k^2 + m^2} \simeq \frac{k_{\max}^4}{16\pi^2}$$

- Taking cutoff to be related to the Planck Mass $k_{\max} = m_p = \sqrt{\frac{1}{8\pi G}}$ $(c = 1 = \hbar)$, gives an energy density of:

$$\langle \rho \rangle \simeq 2 \times 10^{71} \text{ GeV}^4 \implies \Lambda_{\langle \rho \rangle} = 8\pi G \langle \rho \rangle \simeq 4 \times 10^{34} \text{ GeV}^2$$

Problems with the Cosmological Constant

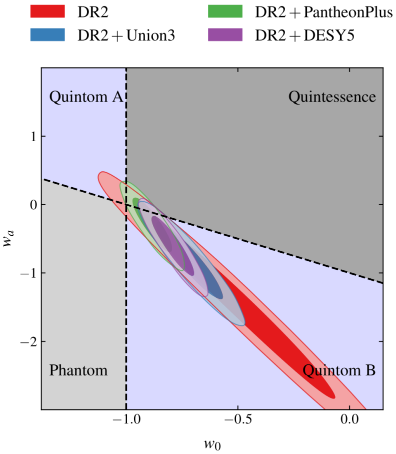
- Lets see how that prediction does, the current measured value of the cosmological constant is:

$$\boxed{\Lambda \simeq 6 \times 10^{-85}\ \mathrm{GeV}^2}$$

- Sometimes this is classed as the worst prediction in physics ever.

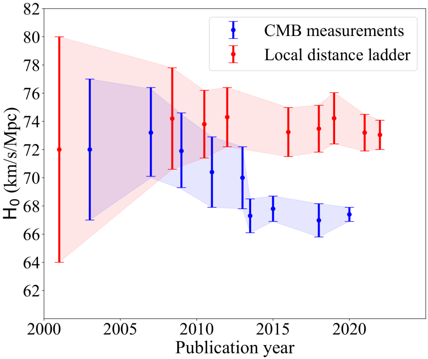
More Problems with the Cosmological Constant

- Λ may not even be constant!
- New DESI data has recently (Sep 2025) been analysed, showing that a constant equation of state is not the best fit of the data.
- Currently the discrepancy is around 2-3 σ



More Problems with the Cosmological Constant

- The Hubble Tension
- The two ways of measure the rate of expansion of our universe today give contradicting results
- Here the tension is at around 5σ.



- Even more problems: Cosmic Co-Incidence problem, why is the energy density of matter $\rho_m \simeq \rho_\Lambda$?

Proposed Explanations of Λ

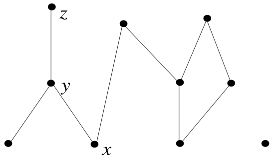
- Quintessence (fifth force): Scalar field that gives rise Dark energy. It can have a dynamical equation of state.
- Modifications of General Relativity such as Massive Gravity.
- Causal Set Theory: see rest of presentation!
- Anthropic principle.

Causal Set Theory

What is Causal Set Theory?

- Causal Set Theory is a discrete approach to Quantum Gravity that postulates spacetime to be fundamentally discrete

- Formally a Causal Set must be:
  1. Reflexive: $x \preceq x$.
  2. Acyclic: $x \preceq y \preceq x \Rightarrow x = y$.
  3. Transitive: $x \preceq y \preceq z \Rightarrow x \preceq z$.
  4. Local Finiteness: $|[x, y]| < \infty$.



- Two key take aways: Causal Sets are discrete and preserve causality.

Why do we want Causal Set Theory?

- The main idea, is that quantum mechanics screams at us that there is a fundamental scale to physics.
- It is widely considered that the notion of spacetime ceases to make sense on scales smaller than the Planck length. But why discrete?
- Discreteness is the most natural solution to the "singularities" or "divergences" that plague a lot of modern physics:
  1. QFT divergences in loop corrections
  2. Curvature singularities at the centre of black holes.
  3. Divergence in entropy of a black hole.
- Causal Sets can alleviate these problems while still treating spacetime in a proper manner.

Can Causal Set Theory Work?

- How can a discrete causal set describe all the intricacies of a continuous spacetime metric $g_{\mu\nu}$?

Theorem: Malament

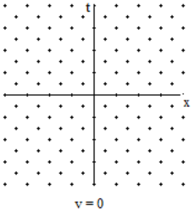
Suppose (M, g) and (M', g') are two distinguishing Lorentzian manifolds and $f : M \to M'$ is a chronological isomorphism, then f is a smooth conformal isometry (meaning $f_* g = \Omega^2 g'$, where $\Omega : M \to \mathbb{R}$).

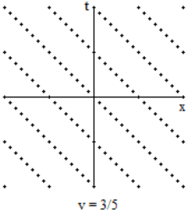
- This means that most of the information about the metric is contained in the causal structure.
- The rest comes from fixing the spacetime volume which Causal Set Theory does by counting elements.

$$\boxed{\text{Order} + \text{number} = \text{geometry}}$$

How are Causal Sets Lorentz Invariant?

- Discrete lattices are not Lorentz Invariant:





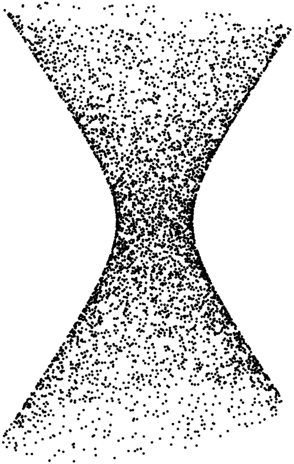
- A Causal Sets can be embedded into a given manifold via a "sprinkling" from a Poisson distribution:

$$P(N, V) = \frac{(\rho V)^N}{N!} e^{-\rho V}$$

- This is further supported by a Theorem by D.Meyer and R.Sorkin that says a causal set sprinkled into (M, g) approaches (M, g) in the infinite density limit.

How are Causal Sets Lorentz Invariant?

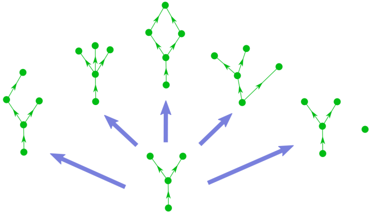
- The Poisson distribution being a function of the spacetime volume V which is invariant under Lorentz transformations.
- This likely the only way of recovering continuous spacetime from a discrete structure.
- Random discreteness saves symmetry.



Everpresent Lambda

Dynamics of Causal Sets

- The most natural way for a causal set to evolve is for it to "grow".
- Currently this is understood very well classically in the form of Classical Sequential Growth (CSG) models.



- A quantum version of this is not as well understood, though it is expected to follow some gravitational path integral formulation, which now becomes a sum over all possible causal sets:

$$Z(V) = \sum_{M} \int \mathcal{D}g \; e^{iS[g]} \to Z(N) = \sum_{|C|=N} e^{iS[C]}$$

- Following the CSG models, N and hence the volume V is kept constant. This means N (or V) plays the role of time.

Uni-modular Gravity

- Uni-modular gravity is a well studied theory of gravity where the volume is held constant and only diffeomorphisms that leave the volume element invariant are allowed.
- This form of gravity can be used to show why the large constants from QFT do not appear in Einstein's Equations.
- In uni-modular gravity there is no inherent cosmological constant, but it arises from imposing that the volume is constant.
- In a rough approximation this leads to a conjugacy between Λ and the volume V such that we can say quantum mechanically:

$$\Delta\Lambda\Delta V \sim \frac{1}{2}$$

- This is analogous to the relation between $\Delta E \Delta t \sim \frac{1}{2}$ from non-relativistic QM.

The Main Argument

- Our assumptions are:
  1. Assumption 1: We keep the volume V fixed from which the unimodular gravity above that tells us $\Delta\Lambda\Delta V \sim \frac{1}{2}$.
  2. Assumption 2: Spacetime is a causal set, obtained from sprinkled using the Poisson distribution into the continous manifold we see.
- The Poisson distribution has two feature of note.
  - The expected number of points in a region of Volume V is $N = \rho V$.
  - The standard deviation of the distribution is $\Delta N = \sqrt{N}$
- These two together mean that a causal set with a fixed number of elements N, there are an ensemble of continuum spacetimes that it can resemble, with mean $V = \rho N$ and standard deviation $\Delta V = \sqrt{V}$.

The Causal Set Prediction

- We make one more assumption:
  3. Assumption 3: The mean value about which Λ fluctuates is 0 so that $\Lambda \sim \Delta\Lambda$ (really we mean $\Lambda \in [-\Delta\Lambda, \Delta\Lambda]$)
- To get an order of magnitude estimate we can put everything together:

$$\Lambda \simeq \Delta\Lambda \simeq \frac{1}{2\Delta V} = \frac{1}{2\sqrt{V}}$$

- Putting this into numbers, we take V to be the volume of the past light cone of our current point, i.e. spatial size of observable universe times the age of the universe. This gives us a predicted magnitude of:

$$\boxed{\Lambda_{CS} \simeq 1 \times 10^{-85}\ \mathrm{GeV}^2}$$

The Timeline of Events

- This argument was first fleshed out by Rafael Sorkin.

[23] R.D. Sorkin, "Spacetime and Causal Sets", in J.C. D'Olivo, E. Nahmad-Achar, M. Rosenbaum, M.P. Ryan, L.F. Urrutia and F. Zertuche (eds.), *Relativity and Gravitation: Classical and Quantum*, (Proceedings of the *SILARG VII Conference*, held Cocoyoc, Mexico, December, 1990), pages 150-173, (World Scientific, Singapore, 1991), and references therein.

- But when was the Cosmological Constant First measured?

THE HIGH-Z SUPERNOVA SEARCH: MEASURING COSMIC DECELERATION AND GLOBAL CURVATURE OF THE UNIVERSE USING TYPE Ia SUPERNOVAE[1]

Brian P. Schmidt,[2] Nicholas B. Suntzeff,[3] M. M. Phillips,[3] Robert A. Schommer,[3] Alejandro Clocchiatti,[3,4] Robert P. Kirshner,[5] Peter Garnavich,[5] Peter Challis,[5] B. Leibundgut,[6] J. Spyromilio,[6] Adam G. Riess,[5,7] Alexei V. Filippenko,[7] Mario Hamuy,[8] R. Chris Smith,[4,9] Craig Hogan,[10] Christopher Stubbs,[10] Alan Diercks,[10] David Reiss,[10] Ron Gilliland,[11] John Tonry,[12] José Maza,[13] A. Dressler,[14] J. Walsh,[6] and R. Ciardullo[15]

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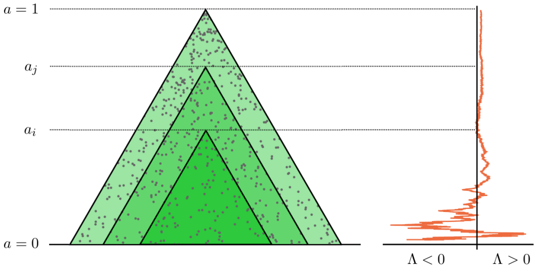
ABSTRACT

- So this was a **pre**diction of the Cosmological Constant! Not a **retro**diction. At the time Λ was believed to be 0.

Everpresent Lambda Cosmologies

- "Everpresent Lambda" models: Λ fluctuates dynamically.
- These models can alleviate the Hubble tension and are capable of producing realizations that fit SN-Ia data better than ΛCDM.
- Named so as Λ is always at a relevant energy scale. (alleviating the coincidence problem).



- In these models Λ can theoretically become negative, so evidence of this would be compelling evidence for Everpresent Lambda.

Thanks for Listening!